

Interference Effect between the Coulomb and Nuclear Excitation Contribution to the Inelastic Scattering of ${}^6\text{Li}$ from ${}^{70}\text{Ge}$ and ${}^{72}\text{Ge}$ at 44 MeV Incident Energy.

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□ ABSTRACT □

We have made an analytical study to differential cross - sections for elastic and inelastic scattering of 44 MeV lithiums beam from ${}^{70,72}\text{Ge}$, over the angular range $\theta_{c.m.} \approx 10^\circ$ to 50° , The measured angular distributions for the 0_1^- ground state and for the first 2_1^+ and 3_1^- inelastic (excited) states obtained for ${}^{70}\text{Ge}$ and ${}^{72}\text{Ge}$ have been analysed within the framework of the collective model using the Distorted Waves Born Approximation DWBA, including coulomb and nuclear excitation. The comparison between the experimental data (θ grazing angle and the experimental angular distributions for the 0_1^+ , 2_1^+ and 3_1^- states) and the results obtained from the analysis give good agreement.

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مفعول التداخل بين التحريض الكولوني والتحريض النووي في التبعثر اللامرن لجسيمات الليثيوم ${}^6\text{Li}$ على نوى الجرمانيوم ${}^{72}\text{Ge}$ بطاقة قذف قدرها 44 مليون إلكترون فولط

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□ الملخص □

قمنا بدراسة تحليلية للمقاطع العرضية التفاضلية للتبعثر المرن واللامرن لحزمة من جسيمات الليثيوم بطاقة قذف قدرها 44 مليون إلكترون فولط على نوى الجرمانيوم ${}^{70}\text{Ge}$ و ${}^{72}\text{Ge}$ ، في المجال الزاوي الذي يتراوح ما بين 10° درجات و 50° درجة. إن الدراسة التحليلية للتوزعات الزاوية التجريبية، للسوية الأساسية 0_1^+ وللسويات الأولى 2_1^+ و 3_1^- المحرصة لكل من ${}^{70}\text{Ge}$ و ${}^{72}\text{Ge}$ ، تمت باستخدام تقريبات بورن في الأمواج المشوهة DWBA المعتمدة على النموذج الجماعي، أخذين بعين الاعتبار التحريض الكولوني والتحريض النووي. إن المقارنة بين النتائج التجريبية (θ زاوية grazing والتوزعات الزاوية التجريبية للسويات 0_1^+ ، 2_1^+ و 3_1^-) والنتائج التي تم الحصول عليها من الدراسة التحليلية تعطي توافقاً جيداً.

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1. Introduction:

When a projectile undergoes elastic scattering, the internal energy of the target is unchanged but the projectile is scattered out of the incident beam in a manner which depends on the interaction between the projectile and the target. Since the nuclei is a quantized system it may exist in one of a discrete spectrum of excited states, each one is characterized by a set of quantum numbers, and the higher states may be excited in the process of inelastic scattering in which the projectile transfers a definite amount of energy to the nucleus. Alternatively, the projectile may be captured and a different particle may be emitted, or the original particle may reappear accompanied by other particles. In each of these processes the residual nucleus is left in a well-defined state. Thus the study of elastic scattering provides information about the nucleus in its normal or ground state, while the study of inelastic scattering and reactions provide information on the existence, location and properties of its excited states. The strength with which these states are excited in different reactions is related to the mode of excitation.

The interaction between particles may be listed in order of decreasing strength as follows: strong or nuclear interaction, electromagnetic interaction, weak interaction, gravitational interaction[1,2,3,4]. The gravitational interaction is completely negligible in the context of nuclear reaction. The weak interaction plays an important role in nuclear physics since it is the cause of the slow decays such as the β -decay of the neutron but does not play a significant role in the processes which are generally classified under the heading of nuclear reactions. We are here concerned with scattering and reactions initiated by projectiles which interact with the target nuclei through the strong and/or electromagnetic interactions.

Heavy ion scattering and reactions have been studied for some time, it is only relatively recent that the interference effect between the coulomb and nuclear excitation contribution to the inelastic scattering process has been reported[5,6,7]. This effect depends on the incident energy and atomic numbers of the projectile and target. The fundamental goal of this work is to examine this effect in $^{70,72}\text{Ge}(^6\text{Li}, ^6\text{Li})^{70,72}\text{Ge}$ inelastic scattering at 44 MeV incident energy.

The present work reports on measurement of the differential cross-sections of the inelastic scattering of ^6Li ions from the even mass stable $^{70,72}\text{Ge}$ isotopes, leaving the target nucleus in the excited $(J^\pi = 2_1^+, 3_1^-)$ states. In addition a comparison is made between the collective model Distorted-Wave Born Approximation (DWBA) to explain the results. The experimental procedures have been reported in earlier publications[8,9,10,11].

2. DWBA Formalism for Inelastic Scattering:

In this formalism the initial state i and the final state f of the nuclei are described by the intrinsic wave functions denoted by Ψ_i and Ψ_f respectively, and the transition amplitude for inelastic scattering from the initial state i to a definite final state f is given by equation, see references[12,13,14,15], as:

$$T_{fi}(\vec{k}_f, \vec{k}_i) = \int d\vec{r} X_f^{*-}(\vec{k}_f, \vec{r}) \langle \Psi_f | V_f | \Psi_i \rangle X_i^+(\vec{k}_i, \vec{r}) \quad (1)$$

where \vec{k}_i and \vec{k}_f are the initial and final momenta of the projectile. $X_i^+(\vec{k}_i, \vec{r})$ and $X_f^-(\vec{k}_f, \vec{r})$ are the distorted wave function for the projectile before and after scattering; these functions are calculated by numeric resolution of the Schrödinger equation with an appropriate spherical optical potential, the signes + and - are related to their asymptotic behaviour. The matrix elements $\langle \Psi_f | V_f | \Psi_i \rangle$ connect Ψ_i with Ψ_f by the effective interaction V_f (non-spheric) which induces the transition; V_f is given by the equation:

$$V_f(r) = U(r) + V_c(r) \quad (2)$$

where $U(r)$ and $V_c(r)$ are the nuclear and coulomb potential respectively.

In the collective model DWBA, the form factor I_i of the l multipolarity excitation is given by equation, see reference[14], as:

$$I_i = A_i^{-1} i^l (2l+1)^{-1/2} \delta_l \left(\frac{dU}{dr} + e^2 Z_1 Z_2 \frac{3}{2l+1} \frac{R_c^{l-1}}{r^{l+1}} \right) \quad (3)$$

where $r > R_c$ (Coulomb radius). For $r < R_c$ the factor R_c^{l-1}/r^{l+1} must be replaced by r^l/R_c^{l+2} . A_i is a geometrical factor, Z_1 , and Z_2 are the atomic numbers of the projectile and target respectively.

For a $J^\pi = 0^+$ target nucleus (our case) the spin J values of the collective excited states are restricted to $J = 1$ (transferred angular momentum) and parity $\pi = (-)^l$. For other excited states, applying the direct interaction rules leads to $J = 1, 1 \pm 1$ and $\pi = (-)^l$ from the DWBA l -value due to $S=0$ or 1 transfer generally permitted in inelastic scattering of nucleons.

3. Analysis of Data:

Nuclear projectiles which are charged, such as proton, α -particles, heavy ions, interact with the nuclei through both the Coulomb and nuclear interactions. For projectiles whose energies are well above the Coulomb barrier (the minimal energy given to projectile to touch the target), in the centre - of mass system, is given by[15,16].

$$\begin{aligned}
B_{c.m} &= \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r} (\text{joule}) = \frac{e^2}{4\pi\epsilon_0} \frac{Z_1 Z_2}{R_0 \left(A_1^{1/3} + A_2^{1/3} \right) + \delta} (\text{joule}) \\
&= \frac{Z_1 \cdot Z_2}{\left(A_1^{1/3} + A_2^{1/3} \right) + 0,7} (\text{MeV}) \quad (4)
\end{aligned}$$

where $\delta = 1$ Fermi, $R_0 = 1.5$ Fermi, $(A_1, Z_1), (A_2, Z_2)$ are the mass numbers and atomic numbers of the projectile and target, both interactions are effective and the scattering amplitudes add coherently and interfere. Since the calculation is made by decomposing the distorted wave functions in partial waves, the precision of calculation depend on l_{\max} which is the number of partial waves used. More, the radial integrals T_{β} are made until a radius $R_{\max} = 50$ fm, we neglect the impact parameters b (the distance between the centre of target and the centre of projectile during the interaction) above R_{\max} . In the inelastic scattering of heavy ions, Coulomb excitation plays a dominant role. This is due to the large $Z_1 Z_2$ product, as well as the long range of the interaction causing the excitation. The Coulomb excitation contribution to the inelastic scattering is included by adding a term proportional to $1/r^{1+1}$, to the form factor (see section 2). As can be seen, the form factor of the Coulomb excitation falls off as r^{-1-1} .

The Coulomb interaction (Coulomb barrier) prevents the projectiles from penetrating the nucleus and experiencing the nuclear interactions. When the potential $V_f(r)$ contain both nuclear (short-range) and Coulomb (long-range) terms, we can say there is a competition between the both interactions. The scattering angle θ (the angle between the final direction of motion of the projectile and the direction of the incident beam - see figure 1) depends on the interactions, in consequence, we define three angular zones delimited by θ_c (deflection angle or classical scattering angles) and θ_g (grazing angle: for $\theta > \theta_g$, the nuclear interaction becomes more important than Coulomb interaction) (see figure 1). As can be seen in figure (1), the effect of the Coulomb interaction is more important than the effect of the nuclear interaction at small angles.

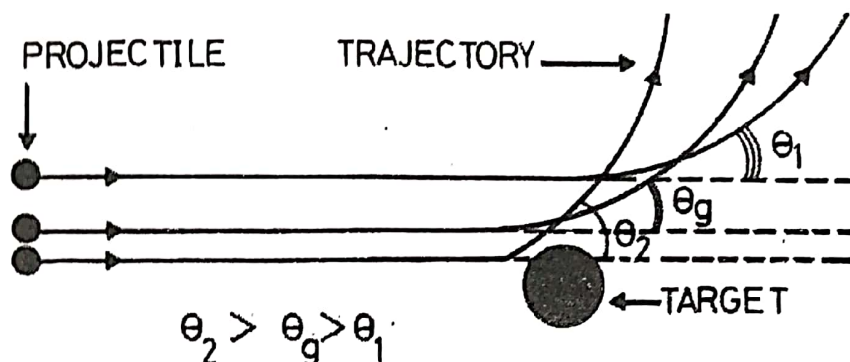


Figure 1.

Figure 1. Definition of the θ_g grazing angle for scattering of a classical particle.

$\theta = \theta_1 < \theta_g \rightarrow$ Coulomb excitation

$\theta = \theta_g \rightarrow$ Coulomb excitation + nuclear excitation

$\theta = \theta_2 > \theta_g \rightarrow$ Coulomb excitation \ll nuclear excitation

When the Z_1 increases ($Z_1 > 2$) the Coulomb interaction, impact parameter, and partial waves increase, while the scattering angle decreases [15,16]. The angular distribution of the reaction is as a function of the scattering angle. It is very important to calculate correctly the inelastic scattering cross section, this means that we must calculate correctly the term Coulomb. In other words, for which value of scattering we can say that the calculation is correct?

For that, investigation were carried out to examine the convergence of the cross section by repeating the calculations using different matching radii (R_{max}), integration step size and number of partial waves (L_{max}) so that θ_c be inferior to smaller angle in the experimental angular range and the Coulomb amplitude be correctly calculated between θ_c and θ_g .

We note that the optical potential used in the present study is defined as the sum of the volume real and volume imaginary nuclear potential and a Coulomb term, thus [12,13,14,15]:

$$\begin{aligned}
 V_f(r) &= U(r) + V_c(r) \\
 &= -V_R f(r, R_R, a_R) - iW_v f(r, R_I, a_I) + V_c(r, R_c)
 \end{aligned}
 \tag{5}$$

where $R_x = r_x A_T^{1/3}$ (A_T is the target mass number, $x=R,I,c$) and $f(r_x) = [1 + \exp(r - R_x)/a_x]^{-1}$ is the standard Woods-Saxon shape function. V_c represents the Coulomb potential of a uniformly charged sphere of radius R_c ,

$$V_c(r) = \begin{cases} (Z_1 Z_2 e^2 / 2R_c) [3 - (r/R_c)^2] & \text{for } r \leq R_c \\ Z_1 Z_2 e^2 / r & \text{for } r \geq R_c \end{cases} \quad (6)$$

We used several parameters of the optical potential in this work[17,18,19,20], the parameters listed in table 1 are referred to as best lithium potential of Cook[17], and we used a collective form factor, relation (3), to describe the interaction between the ${}^6\text{Li}$ projectile and the target nucleus. The form factor is simply related to the derivative of the optical potential used to generate the distorted waves (see, e.g. reference 14 and 21). The calculations of the theoretical differential cross sections have been made in the collective model DWBA[12,13,14,15,21] with the DW4 code[22].

	V_R (MeV)	r_R (fm)	a_R (fm)	W_v (MeV)	r_1 (fm)	a_1 (fm)	r_c (fm)
${}^6\text{Li}+{}^{70}\text{Ge}$	109.5	1.326	0.811	38.88	1.534	0.884	1.3
${}^6\text{Li}+{}^{72}\text{Ge}$	109.5	1.326	0.811	38.43	1.534	0.884	1.3

Table 1. Parameters of the optical potential.

In the angular range of our experiment, 10° - 50° , calculated cross sections are almost similar for the integration steps between 0.07 fermi and 0.13 fermi with a well determined radius. Therefore, a step of 0.1 fermi has been chosen for all the calculations. Figure 2a and Figure 2b show the calculated cross section with the DW4 code[22] for the 2_1^+ states of ${}^{70}\text{Ge}$ as a

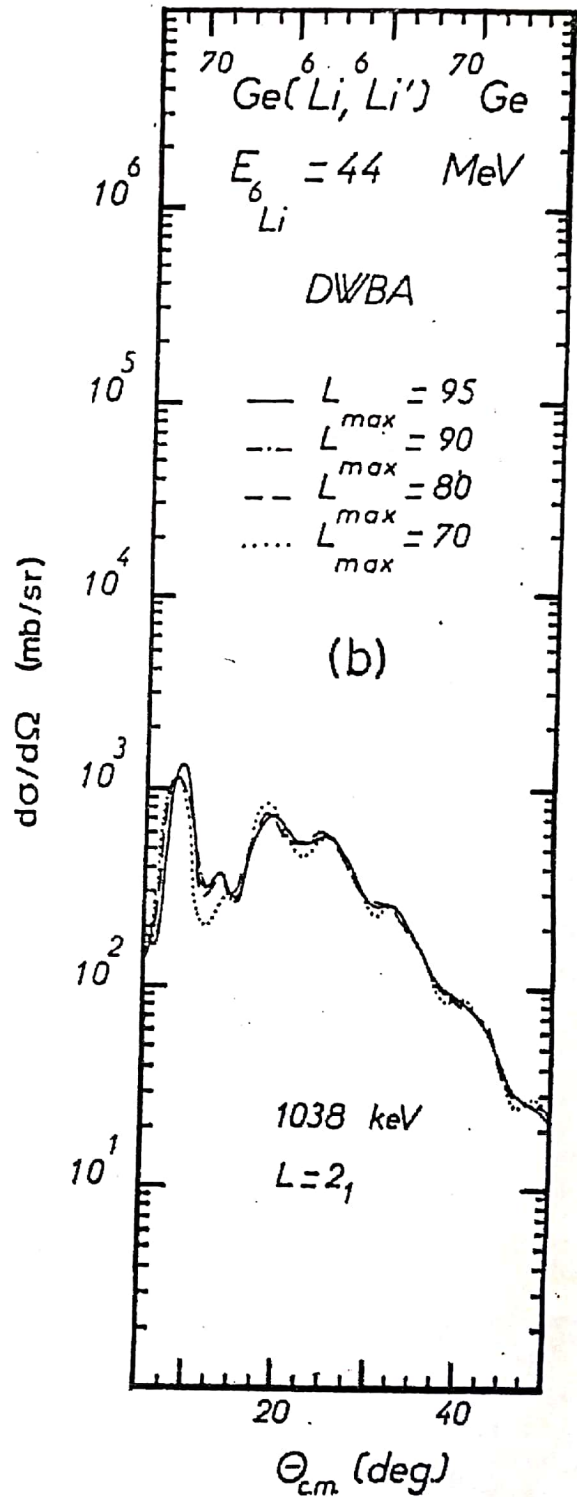
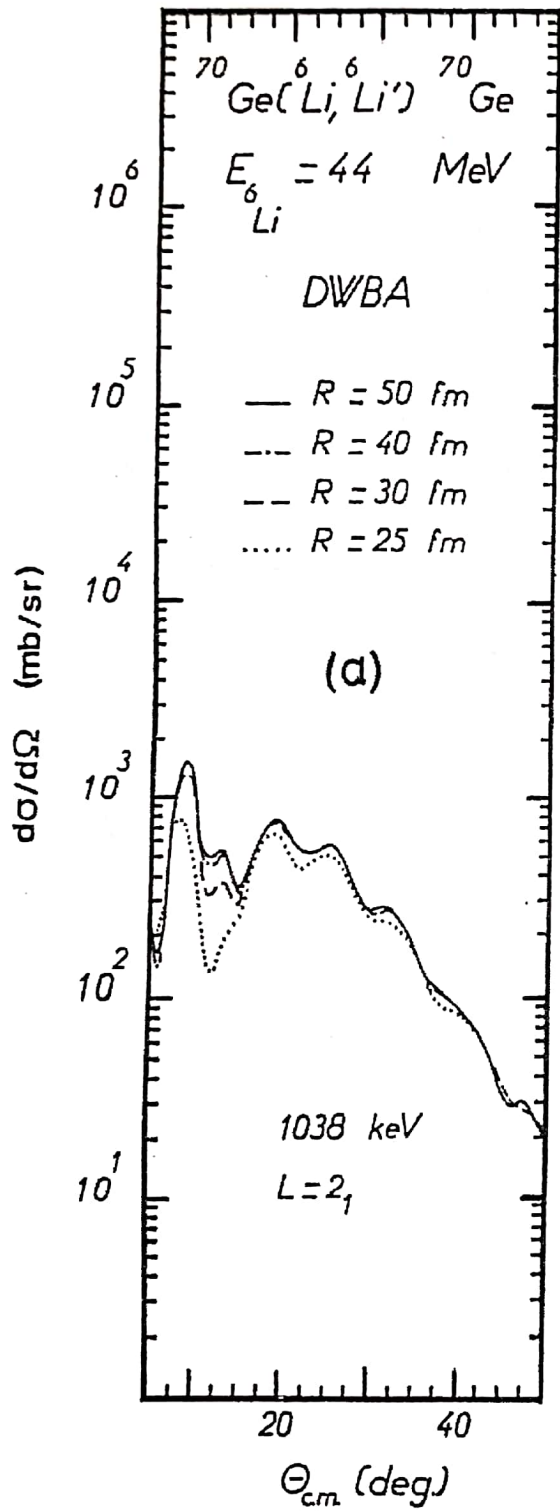


Figure 2. Variation of the calculated 7 cross section for the 2_1^+ state at 1038 KeV excitation energy as a function of R (for L_{max} constant - Fig. 2a) and L_{max} (for R constant - Fig. 2b).

function of R (for L_{\max} constant) and L_{\max} (for R constant) respectively. As can be seen, the calculations converge for $R=40$ fermi and $L_{\max}=90$: both values has been fixed and used in all our calculations. We have chosen the 2_1^+ state because the Coulomb excitation effect is the most important for $l=2$; this effect reduces as a function of l and becomes almost negligible for $l=4$.

In Figure 3 we show separately the contributions of Coulomb and nuclear excitation in the calculated cross section of the 2_1^+ state of ^{70}Ge . As can be seen, the cross section of the Coulomb and nuclear excitation have different compartment and are very oscillatory in particular below of θ_g that we can theoretically calculate, in the centre of mass system, by equation [16] as:

$$\sin \frac{\theta_{g(c.m.)}}{2} = \frac{B_{c.m.}}{2E_{c.m.} - B_{c.m.}} \quad (7)$$

where $B_{c.m.}$ is the Coulomb barrier given by equation (1), $E_{c.m.}$ is the incident energy in the centre - of mass system and related to incident energy in laboratory-system E_1 by the relation: $E_{c.m.} = E_1 \cdot A_2 / (A_1 + A_2)$, by calculating $\theta_{g(c.m.)}$ for the (^6Li , $^6\text{Li}'$) scattering on ^{70}Ge and ^{72}Ge studied at $E_1 = 44 \text{ MeV}$ we find $\theta_{g(c.m.)} \approx 25^\circ$ (for both isotopes), and experimentally we find $\theta_{g(c.m.)} \approx 23^\circ$ (for both isotopes), see figure 3.

We have seen that the importance of Coulomb excitation in the $^{70,72}\text{Ge} (^6\text{Li}, ^6\text{Li}') ^{70,72}\text{Ge}$ inelastic scattering has two immediate consequences:

(i) a large number (90) of partial waves must be included, and (ii) the integrals of the partial waves must be carried out to large radius (40 fermi). As the effect of Coulomb excitation on inelastic scattering is most pronounced at small angles (see figure 1), we examined the estimate of this effect at small angles. To a rough approximation, if L_{\max} is the number of partial waves included in the calculation, then an accurate estimate of the effects of Coulomb excitation is obtained only for scattering angles greater than the classical angle of deflection for a projectile of angular momentum L_{\max} . This angle is given by equation [13,15,21] as: $\theta_c \approx 2n / L_{\max}$ if $n \ll L_{\max}$, where $n = Z_1 \cdot Z_2 e^2 \mu / \hbar^2 k$ in this Coulomb parameter, the reduced mass $\mu = m_1 \cdot m_2 / (m_1 + m_2)$ where m_1 and m_2 are the mass of projectile and target respectively. For $L_{\max} = 90$, for the case of 44 MeV ^6Li incident on ^{70}Ge , we find $n = 5.7$ and $\theta_c = 7.26^\circ$. Consequently, at angles greater than 7.26° our calculations should be accurate.

Figure 4 shows our experimental differential cross sections for the 0_1^+ , 2_1^+ and 3_1^- states together with the best fitting DWBA curves that we have obtained. As can be seen, the DWBA curves agree very well with our data. In all these calculations, the Coulomb (β_1^c) and the nuclear (β_1^N) deformation parameters were adjusted to obtain the fits the data (the square of the deformation parameter is the ratio between experimental and theoretical differential cross sections). The solid curves in figure 4 were obtained with $\beta_1^c = \beta_1^N = \beta_1$; the dashed curve for the 3_1^- state of ^{70}Ge was obtained with $\beta_3^c = 0.75 \beta_3^N$; this leads only to a slight improvement of the fit. The values of the deformation parameters are:

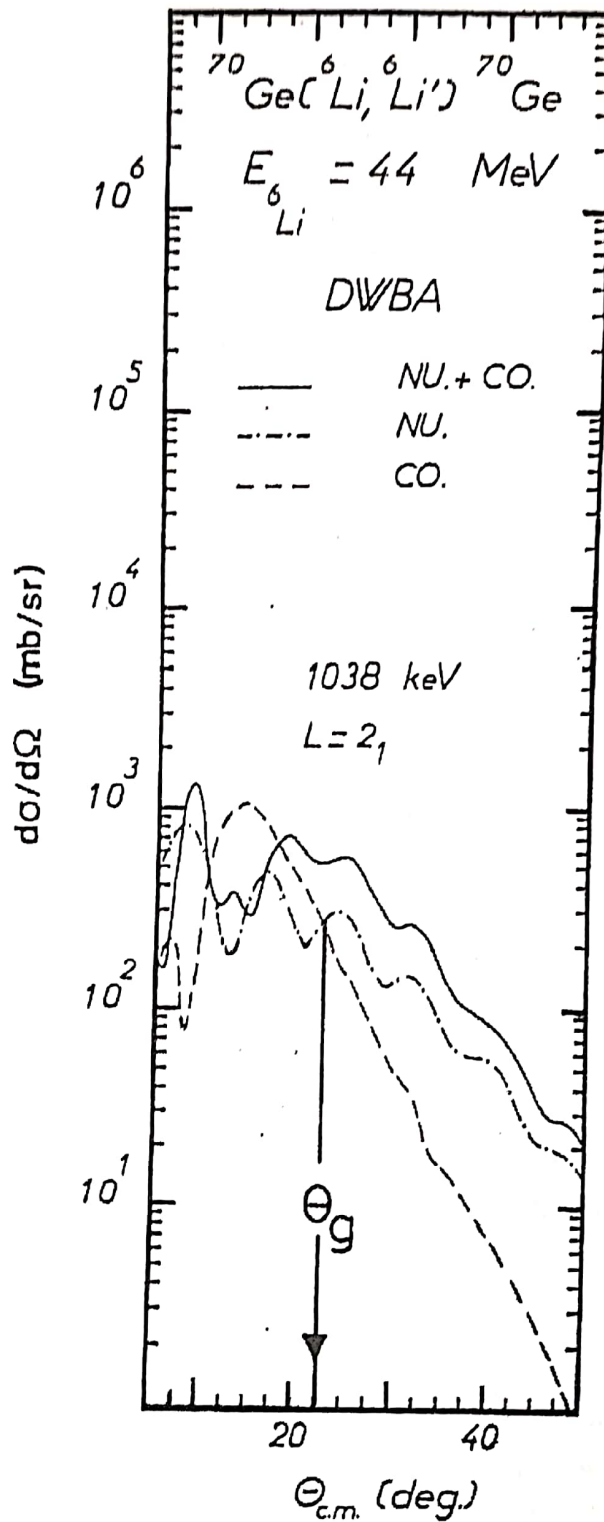


Figure 3. Coulomb excitation (CO) and nuclear excitation (NU) contributions in the calculated differential section of the 2_1^+ state at 1038 KeV. The solid curve represented the calculated differential cross section resulting from the coherently sum of both components: CO and NU.

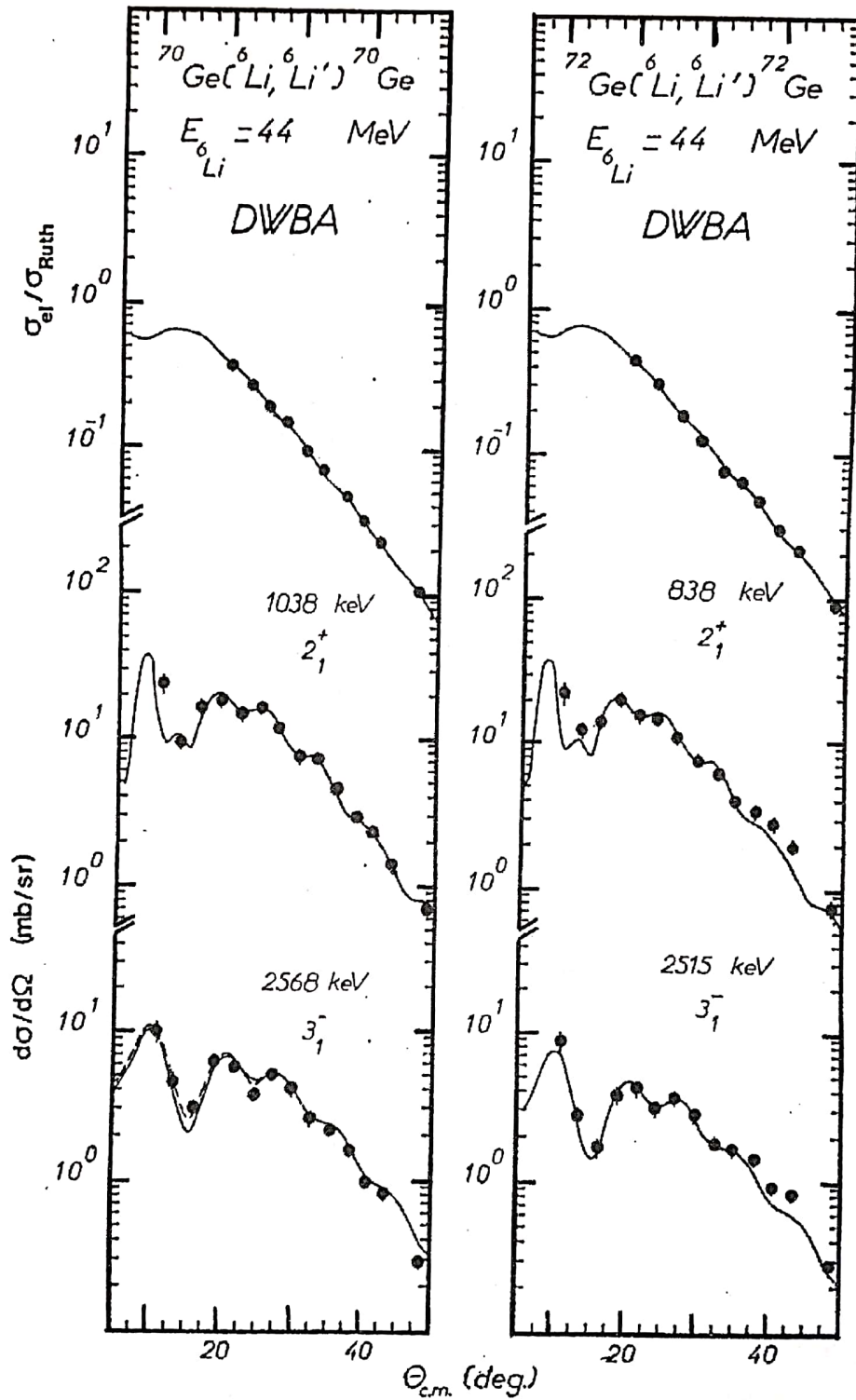


Figure 4. DWBA fits to experimental angular distributions of the 0_1^+ , 2_1^+ and 3_1^- states of ^{70}Ge and ^{72}Ge .

$\beta_2 = 0,18 \pm 0,02, \beta_3 = 0,17 \pm 0,02$ for ^{70}Ge and $\beta_2 = 0,18 \pm 0,02, \beta_3 = 0,14 \pm 0,02$ for ^{72}Ge . We note that the values of the deformation parameters are compared with other values obtained from different works and discussed elsewhere[11].

4. Summary and Conclusions:

We have studied the elastic and inelastic scattering of ^6Li from ^{70}Ge and ^{72}Ge at $E_1 = 44 \text{ MeV}$ incident energy. In spite of this incident energy ($E_{c.m.} \approx 40 \text{ MeV}$) is above the Coulomb barrier ($B_{c.m.} \approx 14 \text{ MeV}$), the Coulomb effect remains important and this is due to the large $Z_1 Z_2$ product. This effect is taken into account by adding a $V_c(r)$ Coulomb potential term to the $U(r)$ nuclear potential in the V_r effective interaction used in the collective model DWBA; in other words, the Coulomb excitation contribution to the inelastic scattering is included by adding a term proportional to $1/r^{l+1}$, to the form factor. In the present work, this effect is negligible for $l \geq 4$.

The study of the separately contributions of Coulomb and nuclear excitation in the calculated cross sections of the 2_1^+ states permitted us to determine $\theta_{g(c.m.)} \approx 23^\circ$; theoretically, we have found $\theta_{g(c.m.)} \approx 25^\circ$. As can be seen, both values are very near.

As the effect of Coulomb excitation on inelastic scattering is most pronounced at small angles, we examined the estimate of this effect at small angles, and we found that our calculations should be accurate for the angles greater than $7,26^\circ$.

Finally, the comparison between our data and the collective model DWBA curves shows a good agreement.

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