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Domination Numbers Of Grids $P_{14} \times P_n$

Mathematics subject classification : 05C69

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□ ABSTRACT □

This paper concerns the domination numbers $\gamma(P_k \times P_n)$ for the $P_k \times P_n$ grid graphs for $k = 14$ and for all $n \geq 1$.

These numbers were previously established for $1 \leq n \leq 12$ [2] , [5].

Keywords: dominating Set, domination number, transformation of a dominating set, Cartesian product of two paths.

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مراتب السيطرة للشبكات $P_{14} \times P_n$

تصنيف موضوع الرياضيات 05C69

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□ ملخّص □

هذه الورقة تهتم بمراتب السيطرة $\gamma(p_k \times p_n)$ لأجل البيانات الشبكية $p_k \times p_n$.
من اجل $k = 14$ ومن اجل كل $n \geq 1$.
هذه المراتب درست من قبل لأجل $1 \leq n \leq 12$ [2]; [5].

الكلمات المفتاحية : مجموعة سيطرة، مرتبة سيطرة، نقل مجموعة سيطرة، الجداء الديكارتي لمسارين.

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Introduction:

A dominating set in a graph is a set of vertices having the property that every vertex not in the set is adjacent to a vertex in the set. The domination number $\gamma(G)$ of a graph G is the cardinality of a smallest dominating set in G.

The problem of finding the domination number of a arbitrary grid graph (=subgraph of $P_k \times P_n$) is NP-complet[3] ,[4].

In this paper, we introduce the concept of transforming the domination

From avert ex in a dominating set D of graph $G = (V, E)$ to avert ex in

$V - D$, where G is a simple connected graph. We give an algorithm using this transformation to obtain a dominating set of agraph G.

A graph $G = (V, E)$ is a mathematical structure which consists of two sets V and E, where V is finite and nonempty, and every element of E is an unordered pair $\{u, v\}$ of distinct elements of V; we simply write uv instead of $\{u, v\}$.

The elements of V are called vertices, while the elements of E are called edges [1].

Two vertices u and v of a graph G are saidto be adjacent if $uv \in E$.

The neighborhood of v is the set of all vertices of G which are adjacent to v;

the neighborhood of v is denoted by $N(v)$. The closed neighborhood of v is $\bar{N}(v)$, $\bar{N}(v) = N(v) \cup \{v\}$.

The degree $d(v)$ of a vertex v is the cardinality $|N(v)|, d(v) = |N(v)|$.

Definition:

Let D be a dominating set of a graph $G = (V, E)$.

1. We define the function C_D, which we call the weight function, as follows: $C_D : V \to \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, $C_D(v) = |\tilde{N}(v)|$,

where $\tilde{N}(v) = \{w \in D : vw \in E \text{ or } w = v\}$

$i.e.$ the weight of v is the number of vertices in D which dominate v.

2. We say that $v \in D$ has amoving domination if there exists a vertex $w \in N(v) - D$ such that $wu \in E$ for every vertex $u, u \in \{x \in N(v) : C_D(x) = 1\}$.

3. We say that avertex $v \in D$ is a redundant vertex of D if $C_D\{w\} \geq 2$ for every vertex $w \in \bar{N}(v)$.

4. If $v \in D$ has a moving domination, we say that v is ine ffi cient if transforming the domination from v to any vertex in $N(v)$ would not produce any redundant vertex.

Grid graph $P_k \times P_n$:

For two vertices v_0 and v_n of a graph G, a $v_0 - v_n$ walk is an alternating se- quince of vertices and edges $v_0, e_1, v_1, \ldots, e_n, v_n$ suchthatconsecutivevertices and edges areincident.

A path is a walk in which no vertex is repeated. A path with n vertices is denoted by P_n, it has $n - 1$ edges; the length of P_n is $n - 1$; the cartesian product $P_k \times P_n$ of two paths is the grid graph with vertex set

$V = \{(i, j) : 1 \leq i \leq k, 1 \leq j \leq n\}$

where $(u_1, u_2)(v_1, v_2)$ isaedgeof $P_k \times P_n$ if $|u_1 - v_1| + |u_2 - v_2| = 1$ [4].

If D is a dominating set of $P_k \times P_n$ which has no redundant vertex, then a vertex $v \in D$ has a moving domination if and only if one of the following two casesoccurs:

Case (1): for every vertex $w \in N(v)$, we have $C_D(w) \geq 2$.

In this case, the domination of v can be transformed to any vertex in $N(v) - D$.

Case(2): there exists exactly one vertex $u \in N(v)$ such that $C_D(u) = 1$. In this case, the domination of v can be transformed only to u.

Analgorithm for finding a dominating set of a graph $P_k \times P_n$ using a transformation of domination of vertices:

1. Let $P_k \times P_n = (V, E)$ be a graph of order greater than 1; $|V| = m$.

2. Let $D = V$ be a dominating set of $P_k \times P_n$.

then, for any vertex $v \in D$ we have $C_D(v) = d(v) + 1 \geq 2$.

3. Pick a vertex v_1 of D, and delete from D all vertices $w, w \in N(v_1)$.

then, for $1 < n < m/2$, pick a vertex $v_n \in D - \cup_{i=1}^{n-1} \bar{N}(v_i)$ and delete from D all vertices $w, w \in N(v_n) - \cup_{i=1}^{n-1} \bar{N}(v_i)$ If D contains a redundant vertex, then delete it.

Repeat this process until D has no redundant vertex.

4. Transform domination from vertices of D which have moving dom- ination to vertices in $V - D$ to obtain redundant vertices and go to step4.

If no redundant vertex can be obtained by a transformation of domination of vertices of D, then stop, and the obtained dominating set D satisfies:

for every $v \in D, \exists w \in \bar{N}(v)$ *such that* $C_D(w) = 1$.

Example:

1. Let (k, n) be the vertex in the $k - th$ row and in the $n - th$ column of the graph $G = P_{14} \times P_{12}$; $|V|$ =168.

2. Let $D = V$, dominating set of G.

3. Pick a vertex $v_1 = (1,2) \in D$, and delete from D all vertices w, $w \in N(v_1)$, then, for $1 < n < 168/2$, pick a vertex v_n, $v_n \in D - \cup_{i=1}^{n-1} \bar{N}(v_i)$, and delete from D all vertices $w, w \in N(v_n) - \cup_{i=1}^{n-1} \bar{N}(v_i)$. We obtain the dominating set D (black circles) in figure 1.

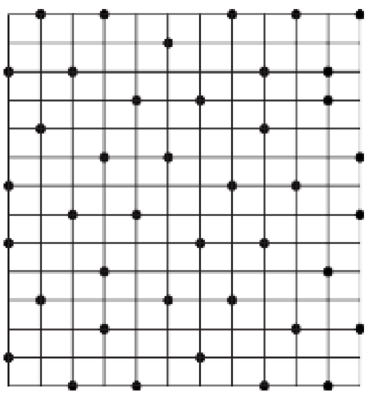


Figure1.

4. Since for every vertex $vD, \exists w \in \bar{N}(v)$ such that $C_D(w) = 1$, D has no red un dantvertices.

5. Transform the domination from the vertex $(1,12)$ to the vertex $(2,12)$ and delete, from D, the resulting redundant vertex$(3,11)$.

The set D indicated in figure 2 (black circles)is a dominating set of $G = P_{14} \times P_{12}$

Note that $D$is a minimum dominating set (see [2]). $\gamma(P_{14} \times P_{12}) = 40$.

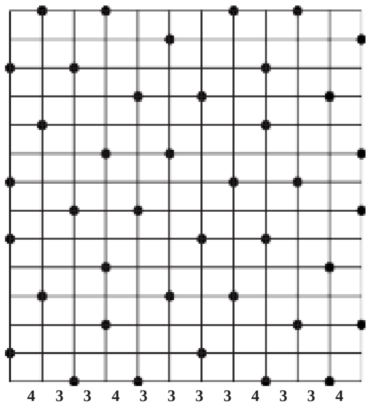
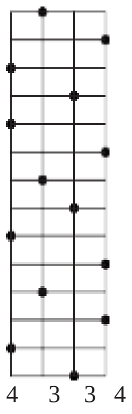
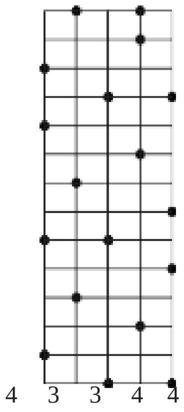


Figure2.

$$n = 12, \gamma(P_{14} \times P_{12}) = 4(n-8) + 24$$
$$= 4n - 8$$

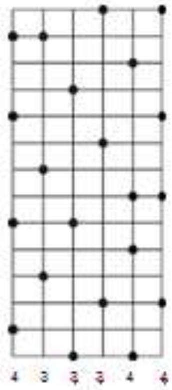
By the same method, we give$\gamma(P_{14} \times Pn)$; $n \geq 1$.



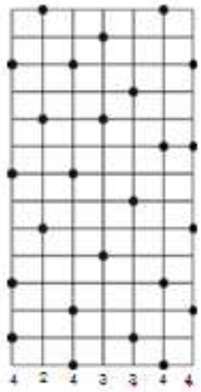


$$n = 4, \gamma(P_{14} \times P_4) = 4(n-2) + 6$$
$$= 4n - 2$$

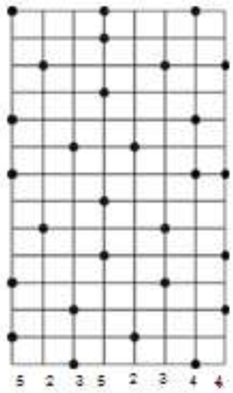
$$n = 5, \gamma(P_{14} \times P_5) = 4(n-2) + 6$$
$$= 4n - 2$$



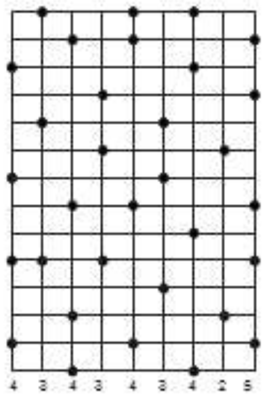
$$n = 6, \gamma(P_{14} \times P_6) = 4(n-3) + 9 = 4n - 3$$



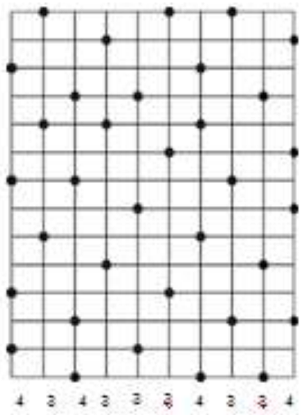
$$n = 7, \gamma(P_{14} \times P_7) = 4(n-3) + 8 = 4n - 4$$



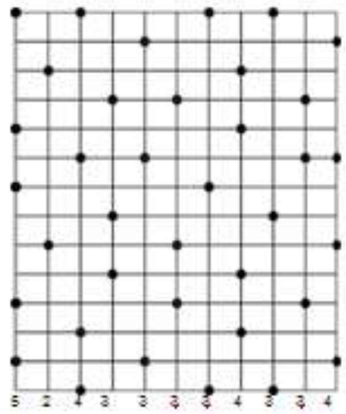
$$n = 8, \gamma(P_{14} \times P_8) = 4(n-6) + 20 = 4n - 4$$



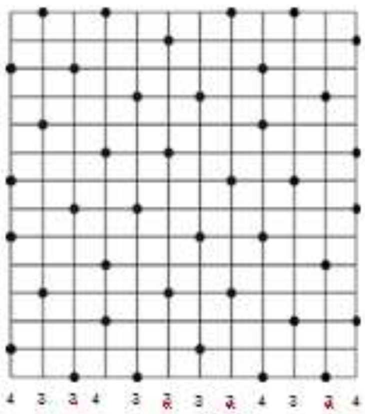
$$n = 9, \gamma(P_{14} \times P_9) = 4(n-5) + 16 = 4n - 5$$



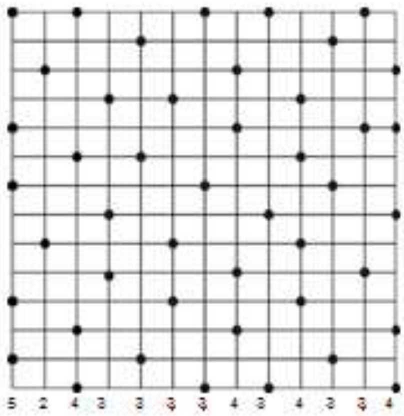
$n=10, \gamma(P_{14}\times P_{10}) = 4(n-6)+18$
$= 4n-6$



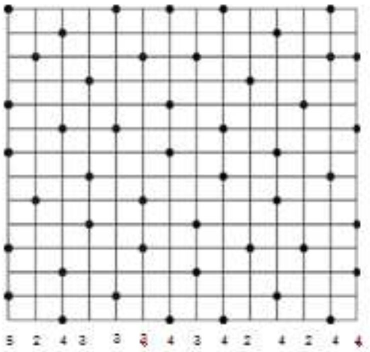
$n=11, \gamma(P_{14}\times P_{11}) = 4(n-8)+25$
$= 4n-7$



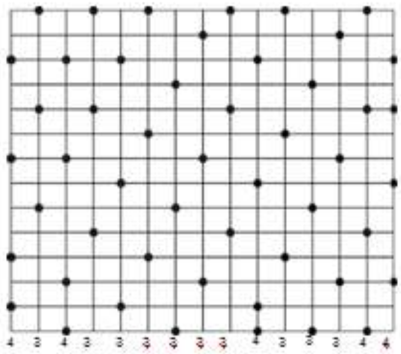
$n=12, \gamma(P_{14}\times P_{12}) = 4(n-8)+24$
$= 4n-8$



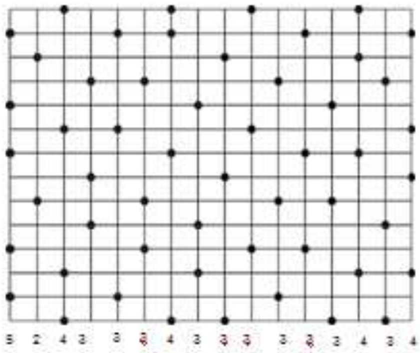
$n=13, \gamma(P_{14}\times P_{13}) = 4(n-9)+28$
$= 4n-8$



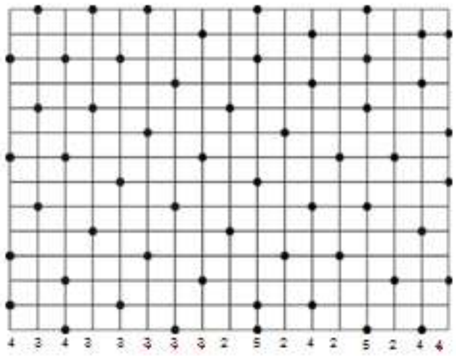
$n = 14, \gamma(P_{14} \times P_{14}) = 4(n-8) + 23 = 4n - 9$



$n = 15, \gamma(P_{14} \times P_{15}) = 4(n-10) + 30 = 4n - 10$



$n = 16, \gamma(P_{14} \times P_{16}) = 4(n-12) + 17 = 4n - 11$



$n = 17, \gamma(P_{14} \times P_{17}) = 4(n-12) + 36 = 4n - 12$

Hence :

$$\gamma(P_{14} \times P_n) = \begin{cases} 5 \,; n = 1 \\ 8 \,; n = 2 \\ 11 \,; n = 3 \\ 14 \,; n = 4 \\ 18 \,; n = 5 \\ 21 \,; n = 6 \\ 4n - (4 + 8t + 18k) \,; n = 7 + 10t + 22k \\ 4n - (m + 4t + 18k) \,; n = 4 + m + 5t + 22k, 4 \le m \le 7. \\ 4n - (8 + 9t + 18k) \,; n = 12 + 11t + 22k \\ 4n - (m + 5t + 18k) \,; n = 6 + m + 6t + 22k, 12 \le m \le 16. \end{cases}$$

where t, kand $m$are positive integers, $0 \le t \le 1, k \ge 0$.

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