

## دراسة S – خاصة أصلية في صف جبر لي

الدكتور أحمد حسان الغصين \*

( قبل للنشر في 2002/1/6 )

### □ الملخص □

هذا البحث مكرس للإجابة على السؤال التالي :

ليكن  $L$  جبر لي فوق حقل مميزه يساوي الصفر ولتكن  $S$  -خاصة أصلية في صف جبر لي .

هل  $S$ -خاصة أصلية تكون دوما مثالية مميزة في الجبر  $L$  ؟

للإجابة على هذا السؤال عرضنا أولاً بعض التعريفات والمبرهنات والتمهيدات الضرورية ومن ثم برهنا أنه اذا كان  $D$

أي اشتقاق في جبر لي  $L$  بحيث أن  $S(L)^n$  أو  $D(S(L))^n$  حيث  $n$  اكبر او تساوي الواحد عندئذ  $S(L)$  أو  $D(S(L))$

وكذلك بينا انه من اجل أي جبر ارتيني  $S(L)$  هي مثالية مميزة في  $L$  . وبعد ذلك اعطينا مثالا يجب على السؤال الطرح ،

إذ ليس من الضروري أن تكون  $S$  دوما مثالية مميزة في  $L$  .

\* أستاذ مساعد في قسم الرياضيات - كلية العلوم - جامعة تشرين - اللاذقية - سورية .

# 1 - Introduction

A subspace  $I$  of a Lie algebra  $L$  is called an ideal of  $L$ , if  $x \in L, y \in I$ , together imply  $[x, y] \in I$ .

A derivation  $D$  in  $L$  is a linear mapping of  $L$  into  $L$  satisfying the following condition:  
 $D(xy) = D(x)y + xD(y)$  for every  $x, y \in L$ . Denote by  $\text{Der}(L)$  the set of derivations in  $L$ . An ideal  $I$  of  $L$  is said to be characteristic ideal of  $L$ , denote it by  $I \triangleleft L$ , if  $I$  is sub vector space of  $L$  and  $D(I) \subseteq I$  for every  $D \in \text{Der}(L)$ . The ideal  $I$  of Lie algebra  $L$  is said to be  $D$ -invariant, if  $D(I) \subseteq I$ ,  $D$  is derivation of  $L$ . Define a sequence of ideal of  $L$  by

$$L^{(0)} = L, L^{(1)} = [L, L], L^{(2)} = [L^{(1)}, L^{(1)}], \dots, L^{(i)} = [L^{(i-1)}, L^{(i-1)}]$$

Called  $L$  solvable if  $L^{(n)} = 0$  for some  $n$

Define a sequence of ideal of  $L$  by  $L^0 = L, L^1 = [L, L], L^2 = [L, L^1], \dots, L^n = [L, L^{n-1}]$

$L$  is called nilpotent if  $L^n = 0$  for some  $n \geq 1$ . [see 1].

A Lie algebra  $L$  is called a complete Lie algebra if its center  $C(L)$  is zero and its derivations are all inner [see 2]. Let  $L$  be finite-dimensional Lie algebra over a field of characteristic zero, then  $L$  has Levi decomposition i.e.  $L = S + R$ , where  $S$  is a maximal semi simple sub algebra of  $L$  and is called the Levi sub algebra of  $L$  and  $R$  is maximal solvable ideal of  $L$  and is called the radical of  $L$ , the ideal

$I_0 = [L, L] \cap R = [L, R]$  is called the nilpotent radical of  $L$  [see 2].

**DEFINITION 1-1:** The class  $S$  of S-algebra is said to be S-radical property in class Lie algebra, if the class  $S$  satisfy the following conditions:

- 1)- Class  $S$  is closed homomorphic,
- 2)- Every algebra  $L$  contain a maximal ideal belongs to class  $S$ , denote it by  $S(L)$ ,
- 3)- For every algebra  $L$ , quotient algebra  $L/S(L)$  no contain ideals difference of zero belongs to class  $S$ .

S-radical property is called over solvable, if solvable algebra's belongs to class  $S$ .

## II - D-INVARIANT RADICAL PROPERTY

All considered algebra in this part are Lie algebra over a field of characteristic zero

**DEFINITION 2-1:** S-radical property is said to be D-invariance if for every algebra  $L$ , its radical is characteristic ideal in  $L$ .

**LEMMA 2-1:** If  $I$  is an ideal of algebra  $L$ , such that  $I^2 = I$ , then  $I$  is characteristic ideal of  $L$ .

**Proof** its clearly.

**LEMMA 2-2:** Let  $S$  is an S-radical property, if there exists non zero S-radical of abelian algebra  $B$ , then  $S$  is a solvable property.

**Proof:** Let  $L$  be an arbitrary nonzero abelian algebra, denote by  $\langle a \rangle$  the ideal which generated by  $a$  in algebra  $L$ . Because  $L$  and  $B$  are abelian algebra's, so every linear mapping between  $L$  and  $B$  is homomorphism algebra, that is ideal  $\langle a \rangle$  is homomorphism image nonzero algebra S-radical  $B$ . Hence, ideal is S-radical its true for every element algebra  $L$ , then  $L$  is S-radical algebra. Further proof is induction on grade solvability algebra.

**LEMMA 2-3:** If  $I$  is S-radical ideal of algebra  $L$ , such that  $L/I$  is S-radical algebra, then algebra  $L$  is S-radical.

**Proof:** [see 1,3].

**THEOREM 2-1:** If S-radical property is not invariance, then  $S$  is radical property over solvable.

**Proof:** Let  $S$  be a radical property which is not invariance on account derivation, then there exists so algebra  $L$  and so it derivation  $D$ , that  $S(L)$  not contain  $D(S(L))$ . By lemma 2-1, we get  $(S(L))^{2-1} S(L)$ . Because algebra  $S(L)$  is  $S$ -radical, then

$B = S(L) / (S(L))^2$  is nonzero  $S$ -algebra. Algebra  $B$  is abelian, hence since lemma 2-1 we have  $S$ -radical property is over solvable.

**LEMMA 2-3:** If  $I$  is ideal of algebra  $L$ ,  $D$  it derivation, then  $I + D(I)$  is also ideal of algebra  $L$  [ see 1 ].

**LEMMA 2-4:** If  $I$  is solvable ideal of algebra  $L$ ,  $D$  it derivation, then  $I + D(I)$  is solvable ideal of algebra  $L$  [see 1 ].

**LEMMA 2-5:** If  $I$  is ideal of algebra  $L$ , then for any derivation  $D$  of algebra  $L$  we get:

$$1)- D(I^{(n+1)}) \subset I^{(n)},$$

$$2)- D(I^{n+1}) \subset I^n.$$

**Proof:** It is known [5], if  $I$  is ideal of algebra  $L$ , then for every natural number  $k > 0$ ,  $I^{(k)}$ ,  $I^k$  are ideals in  $L$ . So we have

$$D(I^{(n+1)}) = D[I^{(n)}, I^{(n)}] \subset I^{(n)} D(I^{(n)}) + D(I^{(n)}) I^{(n)} \subset I^{(n)}$$

Proof to condition 2 is similar.

**THEOREM 2-2:** Let  $S$  be a radical property in class lie algebra, if  $D$  is any derivation of lie algebra  $L$ , that  $D((S(L))^n) \subset S(L)^n$  for some  $n \geq 1$ , then

$$D((S(L))) \subset S(L).$$

**Proof:** Let  $D(S(L)^{(n)}) \subset S(L)^{(n)}$ ,  $n \geq 1$ . Since  $S(L)^{(n)} \subset S(L)$ , then

$S(L_1) = S(L) / S(L)^{(n)}$ , where  $L_1 = L / S(L)^{(n)}$ , from this we have that  $S(L_1)$  is solvable ideal of algebra  $L$ , hence by lemma (2-4)  $S(L_1) + D_1(S(L_1))$  is solvable ideal in

Algebra  $L$ ; where  $D_1$  is derivation  $D$ -algebra  $L$ .

If  $S$  property is not solvable, then by theorem 2-1 we get  $D(S(L)) \not\subset S(L)$ . Assume that the  $S$ -property is over solvable, then  $S(L_1) + D_1(S(L_1)) \subset S(L_1)$  hence  $D_1(S(L_1)) \subset S(L_1)$  thus  $D(S(L)) \subset S(L)$

**Corollary 2-1:** If radical  $S(L)$  algebra  $L$  satisfy condition  $S(L_1)^{(n)} = S(L)^{(n+1)}$  for some natural number  $n$ , then  $S(L)$  is characteristic ideal in algebra  $L$ .

**Proof:** According to the lemma 2-5,  $D(S(L)^{(n+1)}) \subset S(L)^{(n+1)}$  for any derivation  $D$  of algebra  $L$ , from this and from theorem 2-2 we have  $D(S(L)) \subset S(L)$ .

**Theorem 2-3 :** Let  $S$  be a  $S$ -radical property, then for every Artinian algebra  $L$  radical  $S(L)$  is characteristic ideal in  $L$ .

**Proof:** If  $L$  is Artinian algebra, then there exists a natural number  $k$  such, that

$S(L)^{(k)} = S(L)^{(k+1)}$  from this and according corollary 2-1 from theorem 2-1 we have  $D(S(L)) \subset S(L)$  for any derivation  $D$  of algebra  $L$ .

**Corollary 2-2 :** Let  $L$  be an algebra,  $S$ -radical property and  $D$ -derivation algebra  $L$  if linear space  $D(S(L))$  is finite dimensional, then  $D(S(L)) \subset S(L)$ .

**Proof:** Since the linear space  $D(S(L))$  has finite dimensional, therefore there exists a natural number  $n$  such that  $D(S(L)^{(n)}) = S(L)^{(n+1)}$  from lemma 2-5 and corollary 2-1 from theorem 2-2 we get  $D(S(L)) \subset S(L)$ .

Now, we reflect over the following question. If radical algebra  $L$  precipitate as simple component in  $L$  the is it characteristic ideal in  $L$ ?

To this effect we consider the following situation.

Let  $j$  be a any homomorphism algebra  $L$  in algebra  $B$ , we denote by  $L \dot{\cup} B$  the simple sum  $L \dot{\cup} B$  to vector spaces  $L, B$  which defined in following way :

The multiplication operation, if  $a \in L, b \in B$ , then  $a \cdot b = j(a) \cdot b$ . Clearly show that  $L \cap B$  with above operation is Lie algebra over the same field.

In algebras  $L$  and  $B$  easy check, that  $B$  is ideal in algebra  $L \cap B$ .

**Lemma 2-6:** Let  $D$  be a derivation of algebra  $L \cap B$  such, that  $D(B)$  non contain in algebra  $B$ , then  $C(L)$ , (the center of algebra  $L$ ) is different of zero.

**Proof:** From assumption there exists an element  $b \in B$  such, that  $D(b) \notin B$ . Hence  $D(b) = b' + b^*$  where  $b' \in B, b^* \in L, b^* \neq 0$ . Now let  $a$  be a arbitrary element of algebra  $L$ , then  $a \cdot b = j(a) \cdot b \in B^2$ , consequently  $D(a \cdot b) = D(a) \cdot b + a \cdot D(b) \in B$ . Because  $b, b' \in B$ , that  $B \cap D(a \cdot b) = D(a) \cdot b + a \cdot D(b) = D(a) \cdot b + ab' + ab^*$ , since  $D(a) \cdot b + ab' \in B$  then  $ab^* \in B$ . On the other hand  $ab^* \in L$  hence  $ab^* = 0$  that is the Centrum  $C(L)$  of algebra  $L$  contain element  $b^* \neq 0$  as desired.

**Theorem 2-4 :** Let  $S$ -radical algebra  $L \cap B$  be equal to  $B$ , then it is characteristic ideal in algebra  $L \cap B$ .

**Proof:** According to the theorem 2-1 we can assume, that  $S$ -radical property is over solvable. Suppose that for certain derivation  $D$  of algebra  $L \cap B, D(B) \not\subseteq B$  from this and according to lemma 2-6 that in algebra  $L$  there exists nonzero ideal  $I$  Centrum  $C(L)$  algebra  $L$ . Algebra  $I + B$  is  $S$ -radical as homomorphic image

$(L \cap B) / B$ , this is not possible because  $S(L \cap B) = B$ .

**COROLLARY 2-3 :** Let  $S$  be a  $S$ -radical property of algebra  $L$ , if the radical  $S(L)$  Algebra  $L$  distribute as simple component, then it is characteristic ideal algebra  $L$ .

**Proof:** From assumption there exists ideal  $I$  algebra  $L$ , such that  $L = I + S(L)$ .

Let  $j$  be a zero homomorphism algebra  $I$  in  $S(L)$ , then

$L = I + S(L) = I \cap S(L)$ . Now by theorem 2-4  $D(S(L)) \not\subseteq S(L)$  for any derivation  $D$  algebra  $L$ .

**DEFINITION 2-2:** Derivation  $d$  algebra  $L$  is said to be nil-derivation, if for every element  $a \in L$  there exists as natural number  $n > 0$ , such that  $D^n(a) = 0$ .

It is known [4,5] that if  $L$  is nil-derivation algebra  $L$  invariance relative on automorphism algebra  $L, D(L) \subseteq L$ . From this we have the following lemma.

**LEMMA 2-7:** If  $D$  is nil-derivation algebra  $L$ , then  $D(S(L)) \subseteq S(L)$  whereas  $S$  is  $S$ -radical property.

Let  $a_1, a_2, a_3, \dots, a_n$  are elements of algebra  $L$ . Denote by  $(a_1, a_2, a_3, \dots, a_n)_r$  the multiplication  $n$ -elements of part bricks  $r$ , whereas  $r$  part bracket of word  $a_1, a_2, a_3, \dots, a_n$ .

**LEMMA 2-8:** Let  $a_1, a_2, a_3, \dots, a_n$  are elements of ideal  $I$  of algebra  $L$ , then for every derivation  $D$  algebra  $L$  we have

$$(D(a_1), D(a_2), \dots, D(a_n))_r \in I + 1/n! D^n(a_1, a_2, \dots, a_n)_r$$

**Proof:** From inequality which we can fined in [6] we have

$$D^n(a_1, a_2, \dots, a_n) = \sum_{a_1, a_2, \dots, a_n} \binom{n}{a_1} \binom{n-a_1}{a_2} \dots \binom{n-a_1-a_2-\dots-a_{n-2}}{a_n} (D^{a_1}(a_1), \dots, D^{a_n}(a_n))_r \quad (*)$$

Where the sum over all  $a_1, a_2, \dots, a_{n-1}$  such that  $0 \leq a_i \leq n - a_1 - \dots - a_{n-1}$ .

$a_n = n - a_1 - \dots - a_{n-1}, 1 \leq i \leq n-1, D^0(a) = a$ , as  $a_1 + \dots + a_n = a$  and all exponent  $a_i$  are non negative, then  $a_1 = a_2 = \dots = a_n = 1$  or  $a_i = 0$  for some  $i$ . If exponent  $a_i = 0$ , then element

$$(D^{a_1}(a_1), \dots, D^{a_n}(a_n))_r \hat{=} I.$$

Because  $a_i \hat{=} I$  and  $D(a_i) = a_i$  so one expression which all exponent  $a_i$  are non equal to zero, therefore  $(D(a_1), \dots, D(a_n))_r$  is manoeuvrig factor is equal to  $n!$  hence from (\*) we have  $D^n(a_1, \dots, a_n)_r = n! (D(a_1), \dots, D(a_n))_r$ .

Let us recall that an element  $x$  of Lie algebra  $L$  over a field  $K$  is called algebraic if there exists a polynomial  $f(t) \hat{=} K[t]$  depending on  $x$  such that  $f(\text{adx}) = 0$  [see 7].

**DEFINITION 2-3:** Derivation  $D$  algebra  $L$  is called algebraic derivation bounded index, if there exists a natural number  $n > 1$ , such that for every element  $a \hat{=} L$ ,  $D^n(a) \hat{=} \langle a, D(a), \dots, D^{n-1}(a) \rangle$ , where  $\langle a, D(a), \dots, D^{n-1}(a) \rangle$  is sub algebra generated by elements  $a, D(a), \dots, D^{n-1}(a)$

**THEOREM 2-5:** Let  $S$  be a radical property,  $D$  derivation algebraic bounded index of algebra  $L$ , then  $D(SL) \hat{=} S(L)$ .

**Proof:** By virtue of theorem 2.1, we can assume, that  $S$ -radical property is over solvable, from definition of  $D$  there is a natural number  $n$ , such that

$$D^n(a) \hat{=} \langle a, D(a), \dots, D^{n-1}(a) \rangle \quad \text{for some element } a \hat{=} L$$

Let  $a_1, a_2, \dots, a_n$  are any elements from radical  $S(L)$  and let  $a = ((a_1 a_2) a_3 \dots) a_n$ , by virtue of lemma 2.5 we:

$$D^i(a) \hat{=} D^i(S(L)^n) \hat{=} S(L)^{n-i}, \quad i = 1, 2, \dots, n-1$$

So element  $D^{n-1}(a)$  belongs to radical  $S(L)$  algebra  $L$ . By virtue of lemma 2.3  $S(L) + D(S(L))$  is ideal of algebra  $L$ , hence  $S(L) + D(S(L)) / S(L)$  is solvable ideal of algebra  $L/S(L)$  this is, in the presence of over solvable radical  $S$ , that

$$S(L) + D(S(L)) \hat{=} S(L), \text{ from this we have } D(S(L)) \hat{=} S(L).$$

### III-NORMAL RADICAL PROPERTY

Let  $L$  be a Lie algebra over a field  $K$  and let  $B$  always, associative, commutative with unit element  $1$  (over also  $K$ ) as known in [8] that  $L \ddot{\Delta}_K B$  is Lie algebra and algebra  $L$  may be imbedding in algebra  $L \ddot{\Delta}_K B$ .

**LEMMA 3-1 :** If  $D$  is derivation of algebra  $B$ , then the mapping

$$\text{id} \ddot{\Delta}_K d : L \ddot{\Delta}_K B \otimes L \ddot{\Delta}_K B$$

Defined at below:

$$(\text{id} \ddot{\Delta}_K d)(\overset{\circ}{a} a_i \ddot{\Delta}_K p_i) = \overset{\circ}{a} a_i \ddot{\Delta}_K d(p_i)$$

is derivation of algebra  $L \ddot{\Delta}_K B$ . If however  $D$  is nil-derivation, then derivation  $\text{id} \ddot{\Delta}_K d$  is also nil-derivation.

**Proof :** see N. JACOBSON . Lie algebra . Interscience New York 1962 page 158 .

**DEFINITION 3-1 :** We say that  $S$ -radical property is  $B$ -normal if for every algebra  $L$ ,  $S(L \ddot{\Delta}_K B) = I \ddot{\Delta}_K B$ , where  $I$  is reliable ideal of algebra  $A$ . [see 5]

**THEOREM 3-1 :** S-radical property is B-normal iff for every algebra L , from condition  $S(L\check{A}_K B) \neq 0$  result consequence  $S(L\check{A}_K B) \cap A = 0$  .

Now , we show following lemma .

**LEMMA 3-2 :** If for every algebra L there exists nil-derivations  $d_1, d_2, \dots, d_n$  algebra B such that from condition  $S(L\check{A}_K B) \neq 0$  consequence

$$(id \check{A} d_1)(id \check{A} d_2) \dots (id \check{A} d_n) S(L\check{A}_K B) \neq 0$$

then S-radical property is B-normal .

**Proof:** Suppose that  $S(L\check{A}_K B) \neq 0$  then there is element  $0 \neq a \in S(L\check{A}_K B)$  such that  $d_1, d_2, \dots, d_n$  are nil-derivation ,hence from lemma 2-7 and lemma 3-1 we have for  $i=1,2,3.. (id \check{A} d_n)( S(L\check{A}_K B) \cap S(L\check{A}_K B) ) \neq 0$  , therefore

$$0 \neq (id \check{A} d_1)(id \check{A} d_2) \dots (id \check{A} d_n)(a) \in S(A\check{A}_K B) \cap A$$

Thus we have complete the proof .

**THEOREM 3-2:** Let L be a finite dimension algebra , then for every S-radical property , B-normal radical  $S(L\check{A}_K B)$  is invariance respect to derivation algebra  $L\check{A}_K B$  .

**Proof:** S-radical property is B-normal ,then  $S(L\check{A}_K B) = I\check{A}_K B$  , because algebra L has finite dimension there exist a natural number  $n > 0$  that  $I^{(n)} = I^{(n+1)}$  , therefore we get  $(S(L\check{A}_K B))^{(n+1)} = I^{(n+1)}\check{A}_K B^{(n+1)} = (I\check{A}_K B)^{(n)} = (S(L\check{A}_K B))^{(n)}$  Now according to corollary 1 from theorem 2-2 we have demonstration fact .

**COROLLARY 3-1:** Let L be a finite dimension algebra , then for every S-radical property , ideal  $S(L[x_1, x_2, \dots, x_n])$  is characteristic ideal in  $L[x_1, x_2, \dots, x_n]$  where  $L[x_1, x_2, \dots, x_n]$  is polynomial algebra with infinite quantity variable,  $x_1, x_2, \dots, x_n$  ,

**Proof :** It is known that algebra  $L[x_1, x_2, \dots, x_n]$  is isomorphic with algebra  $L\check{A}_K K[x_1, x_2, \dots, x_n]$ , we show that every  $K[x_1, x_2, \dots, x_n]$  radical property is normal . Suppose that  $S(L\check{A}_K K[x_1, x_2, \dots, x_n]) \neq 0$ , then there is nonzero polynomial  $W(x_{a_1}, x_{a_2}, \dots, x_{a_n}) \in S(A\check{A}_K K[x_1, x_2, \dots, x_n])$

It is known that derivation  $d_a(W)(W / x_a)$  for every valued a is nil-derivation algebra  $L[x_1, x_2, \dots, x_n]$  . Now , we take right derivation  $id \check{A} d_a$  we get

$$0 \neq (id \check{A} d_{a_1}) \dots (id \check{A} d_{a_n}) W(x_{a_1}, x_{a_2}, \dots, x_{a_n}) \in S(A\check{A}_K K[x_1, x_2, \dots, x_n]) \cap L$$

Applying lemma 3.2 we have that S radical property is  $K[x_1, x_2, \dots, x_n]$  normal hence from theorem 3-2  $S(L[x_1, x_2, \dots, x_n])$  is characteristic ideal in algebra  $L[x_1, x_2, \dots, x_n]$  .

Now we investigate problem invariable radicals  $L\check{A}_K B$ , where B is a algebra group  $K[G]$  over a field K , G is abelian group .We define following symbol . Let  $x = a_1 g_1 + a_2 g_2 + \dots + a_n g_n \in L\check{A}_K K[G]$  , where  $a_1, a_2, \dots, a_n \in A$ ,  $g_1, g_2, \dots, g_n \in G$  ,denote by  $Supp x$  the set of element  $g_i$  such that  $a_i \neq 0$  .

If  $Supp x = \{ g_1, g_2, \dots, g_n \}$ , then the number  $n = l(x)$  is said to be length element x

**LEMMA 3-3:** Let  $S$  be a radical property,  $L$  algebra,  $G$ -group, if radical  $S(L[G])$  algebra  $L[G]$  contain element  $x$  with length  $n$ , then  $S(L)$  contain element  $y$  which also length  $n$  and  $e \in \text{Supp } y$ , where  $e$  is unit element of group  $G$ .

**Proof:** Take arbitrary element  $z \in S(L[G])$ ,  $g$  any element group  $G$ . Because group  $G$  is abelian then its elements we can effective with operator of algebra

$L[G]$ , therefore from [5,3] we get  $z \in S(L[G])$ .

Now, let element  $x \in L[G]$ ,  $l(x) = n$  and let  $g \in \text{Supp } x$  because  $l(x) = l(x/g)$ ,  $e \in \text{Supp}(x/g)$  and  $x/g \in S(L[G])$  then be enough take  $y = x/g$

**DEFINITION 2-3:** Let  $K$  is a field. Group  $G$  is said to be  $K$ -complete if for every element  $g \in G$ ,  $g \neq e$ , there is a homomorphism  $f$  group  $G$  in multiplication field  $K^*$  such that  $f(g) = 1$ ,  $1$  is unit element of field  $K$ .

**LEMMA 3-4:** Let  $G$  be a complete group, then for every  $S$  radical property and every  $K$ -algebra  $L$ , if  $S(L[G]) \neq 0$ , then  $S(L[G]) \subseteq A \neq 0$ .

**Proof:** Let  $a = \sum_{i=1}^n a_i g_i$  be a nonzero element from radical  $S(L[G])$  with minimal length. By lemma 3-3 we can assume that  $e \in \text{Supp } a$ . Assumption, that  $g_1 = e$  we show that  $l(a) = 1$ . Assumption  $g_2 \neq e$  because group  $G$  is  $K$ -complete, there exists a homomorphism  $f: G \rightarrow K^*$  such that  $l(g_2) \neq 1$ .

We can defined mapping  $\bar{f}: G \rightarrow L[G]$  at follows [see 6].

$$\bar{f}\left(\sum_{i=1}^n b_i g_i\right) = \sum_{i=1}^n b_i f(g_i) g_i, \quad b_i \in L, g_i \in G$$

$f$  is  $K$ -automorphism algebra  $L[G]$ ,  $f(g_2) \neq 1$ ,  $f(g_1) = 1$ . Hence

$$0 \neq \sum_{i=2}^n [a_i - a_i f(g_i)] g_i = a - \bar{f}(a) \in S(L[G])$$

This is not possible, since  $l(a - \bar{f}(a)) < n$  we have  $l(a) = 1$  and  $S(L[G]) \subseteq L \neq 0$ .

**COROLLARY 3-3:** If group  $G$  is  $K$ -complete, algebra  $L$  has finite dimension, then for every  $S$ -radical property the radical  $S(L[G])$  is characteristic ideal in algebra  $L[G]$ .

**Proof:** The proof is result from corollary 3-1 and theorem 3.2.

Now we give some example for  $S$ -radical property which is not invariable property.

P. M. Gohn [5], gave example Lie algebra in which equation  $a \cdot x = b$  has solution for every  $a \neq 0$ ,  $a \in L$  and for every  $b \in L$ .

We consider algebra

$$R[[x]] = \left\{ \sum_{i=1}^{\infty} a_i x^i \quad ; a_i \in L \right\}$$

It is known that algebra of above formal series  $R[[x]]$  with coefficient from Lie algebra and set  $I$  formula series from  $\sum a_i x^i$ ,  $a_i \in L$  is ideal in  $R[[x]]$ .

We find all image homomorphic of algebra  $I$ .

Let  $0 \neq a \in L$  and let  $b$  be any element from ideal  $I$ , then

$$\begin{aligned}
a &= a_n x^n + a_{n+1} x^{n+1} + \dots + a x^m + \dots, \quad a_n \neq 0 \\
b &= b_1 x + b_2 x^2 + b_3 x^3 + \dots \\
a \cdot b &= c = c_{n+1} x^{n+1} + c_{n+2} x^{n+2} + \dots + c_{n+k} x^{n+k} + \dots \\
c_{n+1} &= a_n b_1 \\
c_{n+2} &= a_n b_2 + a_{n+1} b_1 \\
&\cdot \\
&\cdot \\
&\cdot
\end{aligned}$$

$$c_{n+k} = a_n b_k + a_{n+1} b_{k-1} + \dots + a_{n+k-1} b_1$$

Because in algebra  $L$ , inequality  $ax = b$  has solution for  $a \neq 0$ ,  $a \in L$  and for every  $b \in L$ , choose satisfactory accordingly coefficient  $b_1, b_2, \dots, b_n, \dots$ . We can get any value  $c_{n+1}, c_{n+2}, \dots, c_{n+k}, \dots$  from this result that  $I^{n+1} \subseteq \langle a \rangle$ ;

$\langle a \rangle$  is an ideal generated by element  $a$  from algebra  $L$ .

Now, let  $I/Y$  be any homomorphic image of algebra  $I$ , let  $J \neq 0$  then there exists

$0 \neq a \in J \subseteq I$  from this result there exists a natural number  $m > 0$  such that

$I^m \subseteq \langle a \rangle \subseteq J$ . We showed that every proper homomorphic image of algebra  $I$  is nilpotent.

Let  $S$  be a minimum  $S$ -radical property, such that algebra  $I$  and all its homomorphic image belongs to class  $S$ , (by [8] this radical property exists), we show that  $S(R[x]) = I$ . Clearly  $S(I) = I$  and quotient algebra  $R[[x]]/I$  is isomorphic to algebra  $L$ .

Algebra  $L$  non contain maneuverability ideals and is not nilpotent algebra, also algebras  $L$  and  $I$  are not isomorphic because  $L^2 = L$  but  $I^2 \neq I$  to mean that algebra  $L$  no contain nonzero  $S$ -ideals, hence  $S(R[[x]]) = I$ .

Now let  $a = a_0 + a_1 x^1 + a_2 x^2 + \dots + a_n x^n$  be an element of algebra  $R[[x]]$ . Mapping  $D(a_0 + a_1 x + \dots + a_n x^n) = a_1 + 2a_2 x + \dots + na_n x^{n-1}$  is a derivation of algebra  $R[[x]]$ . Let  $a \neq 0$  be an element of algebra  $L$ , because

$S(R[[x]]) = I$  then  $ax \in I$ , but for derivation  $D$  we have

$$D(ax) = a \in I = S(R[[x]]).$$



## REFERENCES :

.....

- 1 - N. JACOBSON . Lie algebra .Interscience New York 1962
- 2 - J.Cuipo and M. Doj. The Clasification Of Compacte Lie Algebra With Commutative Nilpotent Radical .Pro.of the Am.Math.Soci.V.126 n1 (19980,15-23 .
- 3- N.Divinski, A.sulinski. Kurosh Radical Of Rig's With Opertors .Canad.J.Math 17,(1965), 228-280 .
- 4 -T.Anderson .Hereditary Radical And Derivation Of Algeb.Can.J.Math.21(1968),372-377.
- 5 - P.M. Cohn. On A Class Of Simple Ring's .J.Math.5 (1958) 103-107 .
- 6 - A.M.Slinco Uwagi O Rozniczkowania Pierscieni. Sib.Math.Z.13 (1972) 1395-1397.
- 7 - B.Cuarto,J.Gate. Linearly Compact Algebraic Lie Algebras And Coalgebraic Lie Coalgebras .Proc.of Am.Math.Soci.V.125.N7 (1997), 1945-1952 .
- 8 - A.Kurosh . Radykaly Pierscini I Algebr . Mat Sb.33 (1953) 13-20 .

## Calculation of the photoelectric cross-section for subshells P-compounds

Dr.Bader Al-aaraj\*

(Accepted 18/9/2002)

### □ ABSTRACT □

The intensity of the photoelectron peaks depends on a number of factors including the photoelectric cross-section, the electron escape depth, the spectrometer transmission, surface roughness or inhomogeneties.

Therefore, this work aims at calculating the photoelectric cross-section for different subshells of phosphorus –compounds.

By determining cross-section we could estimate the probability per incident photon for creating a photoelectron in a subshell.

---

\* Associate Professor, Department of Physics, Faculty of Science, Tishreen University, Lattakia-Syria