مجلة جامعة تشرين للدراسات والبحوث العلمية _ سلسلة العلوم الأساسية المجلد (25) العدد (13)2003
Tishreen University Journal for Studies and Scientific Research- Basic Science Series Vol (25) No (13) 2003

The Quadrupole Moments of the Deformed Nuclei in the P-Shell

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(Accepted 7/1/2003)

ABSTRACT

The quadrupole moments of the deformed nuclei in the p-shell: ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}C, and ^{14}N are calculated as functions of the total spin I and the deformation parameter β by assuming that these nuclei have axes of symmetry. Moreover, the single-particle wave functions of a nucleon in a deformed non-axially symmetric nuclei are used to calculate the matrix elements of the quadrupole moment operator. Accordingly, the quadrupole moments of the deformed nuclei in the p-shell ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}C, and ^{14}N are calculated as functions of the deformation parameter β, the non-axiality parameter γ, and the oscillator parameter $\hbar\omega_0^0$, which is obtained as function of the mass number A.

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Tishreen University Journal for Studies and Scientific Research- Basic Science Series Vol (25) No (13) 2003

عزوم رباعي القطب للأنوية المشوهة في الطبقة P

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(قبل للنشر في 2003/1/7)

□ الملخّص □

حسبت عزوم رباعيات الأقطاب الكهربائية للنوى المشوهة المتناظرة محورياً ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}C, و ^{14}N كتوابع للسبين الكلي I ومعامل التشوه β. إضافة إلى ذلك، استخدمت التوابع الموجية للجسيم المفرد (نكليون)طبقاً للنموذج الطبقي في النوى المشوهة وغير المتناظرة محورياً في حساب العناصر المصفوفية لمؤثر عزوم رباعيات الأقطاب الكهربائية وكذلك تم حساب عزوم رباعيات الأقطاب للنوى المشوهة التالية ^{11}B, ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Be, ^{10}B ^{12}C و ^{14}N المشوهة والغير متناظرة محورياً في الطبقة p كتوابع لوسيط التشوه β ووسيط اللامحورية γ ووسيط الهزاز التوافقي $\hbar\omega_0^0$، والذي عبرنا عنه بدلالة العد د الكتلي A.

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1. INTRODUCTION

The nuclear collective motion [1] is a topic of the nuclear structure theory which has grown steadily both in the sophistication of its theory and in the range of data to which it relates. The most central parameters of collective rotation are the moments of inertia [2,3,4] and the quadrupole moments [5] of deformed nuclei. Consequently, the investigations of the nuclear moments of inertia and the quadrupole moments are sensitive checks for the validity of the nuclear structure theories.

The axially symmetric harmonic oscillator potential with the spin-orbit coupling term and the term proportional to the square of the orbital-angular momentum quantum number of the nucleon is often used as a model of the nuclear average field. Having the nilsson's considerations [6], the axially symmetric harmonic oscillator characterized prolate shapes [7]. It is therefore of interest to extend the applicability of the asymmetric model to calculate the energy eigenvalues and eigenfunctions for the possible regions of deformation. Accordingly, the single-particle energy eigenfunctions of a nucleon in a deformed nuclear field with no axis of symmetry are used to calculate the nuclear quadrupole moment.

in the present paper we have calculated the quadrupole moments of the deformed nuclei in the p-shell: ^{6}li, ^{7}li, ^{8}li, ^{9}be, ^{10}b, ^{11}b, ^{12}c, and ^{14}n, by assuming that these nuclei have axes of symmetry. Furthermore, we have used the single-particle wave functions of the asymmetric rotor to calculate the quadrupole moments of the nuclei ^{6}li, ^{7}li, ^{8}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}C, and ^{14}N as functions of the deformation parameter β, the non-axiality parameter γ, and the oscillator parameter $\hbar\omega_0^0$, which is obtained as function of the mass number A, the number of protons Z and the number of neutrons N.

2. The Quadrupole Moment for The Axially Deformed Nuclei

Assuming a charge distribution in accordance with the Thomas-Fermi statistical model applied to the oscillator potential one obtains the intrinsic quadrupole moment, to the second order in the deformation parameter δ [6]

$$Q_0 = 0.8ZeR^2\delta(1+\frac{2}{3}\delta) \quad (2.1)$$

where Z is the number of protons and R is to be taken equal to the radius of charge of the nucleus or $R_Z \approx 1.2\ A^{1/3}$ fm, where A is the mass number.

The relation between the measured quadrupole moment, denoted by Q_S, and Q_0 is given by [8]

$$Q_s = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0 \quad (2.2)$$

where I is the spin-quantum number of the specified nuclear state and K is its component along the body-fixed Z-axis. It turns out that the ground state spin of the nucleus is always $I_0 = \Omega = K$, where Ω is the z-component of the total angular momentum **J**, except when $\Omega = \frac{1}{2}$, in which case the ground state spin I_0 is given as function of the decoupling factor a, as given by Table-III of reference [6]. The decoupling factor a, is determined from the expression of the rotational energy for odd-A nuclei, with $\Omega = \frac{1}{2}$, as follows [8]

$$E_{rot} = \frac{\hbar^2}{2\Im}[I(I+1) + a(-1)^{I+\frac{1}{2}}(I+\frac{1}{2})], \qquad (2.3)$$

where $\Im$ is the nuclear moment of inertia [3].

Another formula for the measured quadrupole moment, Q_s, is given by Greiner and Maruhn [5] as follows

$$Q_s = Q_0 \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}(1+\alpha), \qquad (2.4)$$

Where α is given in terms of the deformation parameter β as follows

$$\alpha = \frac{4}{7}\sqrt{\frac{5}{\pi}}\beta, \qquad (2.5)$$

and the intrinsic quadrupole moment Q_0 is given by

$$Q_0 = \frac{6}{\sqrt{5\pi}} ZeR_Z^2\beta. \qquad (2.6)$$

3. THE SINGLE PARTICLE WAVE FUNCTIONS

For a quadrupole deformation, the equation for the surface of a deformed nucleus is given by [8]

$$R = R_0[1 + \sum_{\mu} \alpha_{2,\mu} Y_{2,\mu}(\theta,\varphi)], \qquad (3.1)$$

where R_0 is the radius of the sphere having the same volume and $Y_{2,\mu}$ are the spherical harmonic functions. If the body-centered frame was selected as the principal axes, we have

$$\alpha_{2,2} = \alpha_{2,-2} = \frac{1}{\sqrt{2}}\beta \sin\gamma,\ \alpha_{2,1} = \alpha_{2,-1} = 0, \alpha_{2,0} = \beta\cos\gamma,$$

where β is the deformation parameter and γ is the non-axiality parameter.

If we suppose that the density of the deformed nucleus can be ideally represented by an ellipsoidal distribution, then it follows that the average potential should also be ellipsoidal. This is most easily achieved by using the anisotropic oscillator as average field. Adding a spin-orbit term and a term proportional to the square of the orbital-angular momentum of the nucleon, to produce the experimental single-particle energy levels, the Hamiltonian operator of a nucleon in a deformed non-axial nucleus is then given by [7]

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + \frac{\mu}{2}\omega_0^2 \mathbf{r}^2 + C\mathbf{l}.\mathbf{s} + D\mathbf{l}^2 - \mu\omega_0^2\beta\cos\gamma\,\mathbf{r}^2\,\mathbf{Y}_{2,0}(\theta,\phi)$$
$$-\frac{\sqrt{2}}{2}\mu\omega_0^2\beta\sin\gamma\, r^2\left(Y_{2,2}(\theta,\phi)+Y_{2,-2}(\theta,\phi)\right) \qquad (3.2)$$

The first four terms in this Hamiltonian represent the spherical case while the first five terms represent the axially-symmetric case. The frequencies ω_x, ω_y and ω_z of the anisotropic oscillator are related to the frequency ω_0 by [7]

$$\omega_k = \omega_0[1-\sqrt{\frac{5}{4\pi}}\beta\cos(\gamma-\frac{2\pi k}{3})], \qquad k=1,2,3 \qquad (3.3)$$

where 1 stands for x, 2 stands for y, and 3 stands for z.

The frequency ω_0 is given in terms of the non-deformed frequency ω_0^0 by [9]

$$\omega_0 = \omega_0(\delta) = \omega_0^0(1-\frac{4}{3}\delta^2-\frac{16}{27}\delta^3)^{-\frac{1}{6}}, \qquad \delta = \frac{3}{2}\sqrt{\frac{5}{4}}\beta.$$

The single-particle wave functions, which are the eigenfunctions of the Hamiltonian operator H, can be obtained by diagonalizing the matrix of the Hamiltonian consisting of the first five terms with respect to the basis functions which are the eigenfunctions of the Hamiltonian consisting of the first four terms and then applying the stationary non-degenerate perturbation method for the last term in equation (3.2), the perturbed term. The single-particle wave functions are then written in the form [7]

$$\psi_i = \left|\Omega^\pi\right\rangle_i = \sum_{j\neq i}\left|N,\Omega^\pi\right\rangle_j . \qquad (3.4)$$

The functions $\left|N,\Omega^\pi\right\rangle$, which represent the axially symmetric case, are expanded in the form of linear combinations of wave functions, which represent the spherically symmetric shape of the nucleus, as follows

$$\psi^i_{N\Omega\pi} = \left|N,\Omega^\pi\right\rangle_i = \sum_{\mathbf{l},\Lambda,\Sigma} C_i^{N\Omega\pi}\left|N,\mathbf{l},\Lambda,\Sigma\right\rangle_i . \qquad (3.5)$$

where $C_i^{N\Omega\pi}$ are the expansion coefficients and $\Omega = \Lambda + \Sigma$ is the z-component of the nucleon total angular momentum vector **j** and $\pi = (-1)^{\mathbf{l}}$ defines the parity of the state. The functions $\left|N\mathbf{l}\Lambda\Sigma\right\rangle$ are given by [2]

$$\left|N\lambda\Lambda\Sigma\right\rangle = a_0^{-\frac{3}{2}}\sqrt{\frac{2\Gamma(\frac{N-\lambda+2}{2})}{[\Gamma(\frac{N+\lambda+3}{2})]^3}}\;\mathbf{e}^{-\frac{\rho^2}{2}}\rho^{\lambda}L^{\lambda+\frac{1}{2}}_{\frac{N-\lambda}{2}}\left(\rho^2\right)Y_{\lambda,\Lambda}(\theta,\varphi)\chi_{s,\Sigma} \quad (3.6)$$

where $\rho = \frac{r}{a_0}, a_0 = \sqrt{\frac{\hbar}{m\omega_0}}, N = 0,1,2,3,....,7$ and $\lambda = N, N-2,...,0 or 1$. The functions $L^{\lambda+\frac{1}{2}}_{\frac{N-\lambda}{2}}(\rho^2)$ are the Laguerre polynomials and $\chi_{s,\Sigma}$ are the single–particle spin wave functions. More details about the construction of the single–particle wave functions ψ_i, equations (3.4) and (3.5), can be found in reference [2].

4. The Quadrupole Moment for the Non-Axially Deformed Nuclei

The intrinsic quadrupole moment of a nucleus consisting of Z protons is given by

$$Q_0 = \sum_{i=1}^{Z} Q_i \;, \quad (4.1)$$

where the single–particle operator Q_i is given by [5]

$$Q_i = e\sqrt{\frac{16\pi}{5}}\int\left|\psi^{i}_{N\Omega\pi}\right|^2 r_i^2 Y_{2,0}(\theta_i,\varphi_i)d\tau \quad (4.2)$$

Carrying out the integration in equation (4.2) with respect to the basis functions $\left|N\lambda\Lambda\Sigma\right\rangle$, equation (3.6), one then obtains

$$Q_i = e\sqrt{\frac{16\pi}{5}}\left\langle N'\lambda'\right|r^2\left|N,\lambda\right\rangle_i\left\langle\lambda'\Lambda'\right|Y_{2,0}\left|\lambda,\Lambda\right\rangle_i . \quad (4.3)$$

The matrix elements of the spherical harmonic operator $Y_{2,0}$ are given by [2]

$$\left\langle\lambda'\Lambda'\right|Y_{2,0}(\theta,\phi)\left|\lambda\Lambda\right\rangle = (-1)^{\Lambda'}\sqrt{\frac{5(2\lambda'+1)(2\lambda+1)}{4\pi}}\begin{pmatrix}\lambda' & 2 & \lambda\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\lambda' & 2 & \lambda\\ -\Lambda' & 0 & \Lambda\end{pmatrix}, \quad (4.4)$$

where the last two terms in (4.4) are 3j-symbols of the rotational group R_3. The matrix elements of the operator r^2 are given by [2]

$$\left\langle N'\lambda'\right|r^2\left|N,\lambda'\right\rangle = (N+\frac{3}{2})\delta_{N'N} + \sqrt{n'(n'+\lambda'+\frac{1}{2})}\delta_{N',N-2} + \sqrt{n'(n'+\lambda'+\frac{1}{2})}\delta_{N',N+2} ,$$

$$\langle N'\lambda'|r^2|N,\lambda'-2\rangle = 2\sqrt{(n'+1)(n'+\lambda'+\frac{1}{2})}\,\delta_{N'N} + \sqrt{n'(n'-1)}\,\delta_{N'N-2} + \sqrt{(n'+\lambda'+\frac{1}{2})(n'+\lambda'-\frac{1}{2})}\,\delta_{N'N+2}$$

$$\langle N'\lambda'|r^2|N,\lambda'+2\rangle = 2\sqrt{(n'+1)(n'+\lambda'+\frac{5}{2})}\,\delta_{N'N} + \sqrt{(n'+\lambda'+\frac{5}{2})(n'+\lambda'+\frac{3}{2})}\,\delta_{N'N-2} + \sqrt{n'(n'-1)}\,\delta_{N'N+2} \quad (4.5)$$

where $N = 2n + \lambda$.

Filling the single–particle wave functions (3.4) for a given nucleus in a definite state and determining the state–expansion coefficients of equation (3.5) it is then possible to calculate the quadrupole moment of the specified nucleus by calculating the necessary matrix elements of equations (4.4) and (4.5).

5. RESULTS AND CONCLUSIONS

The adopted treatment makes it possible to calculate the electric quadrupole moment for axially–symmetric as well as for non-axially symmetric deformed nuclei. Since there are no definite evidences that one of the considered p-shell deformed nuclei has not an axis of symmetry it is then better to calculate the quadrupole moments of these nuclei by assuming that they have axes of symmetry, $\gamma = 0°$, and then repeat the calculations by assuming that these deformed nuclei do not have such symmetry axes, $\gamma \neq 0°$. Comparing the obtained results with the corresponding experimental values it is, then, possible to know whether or not these nuclei bosses axes of symmetry.

In Table–1 we present the calculated values of the electric quadrupole moments of the nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}C, and ^{14}N, according to formula (2.4) for the axially-symmetric case and also formulas (2.4) and (4.1) for the non-axial case. In Table-1 we present also the corresponding experimental values [10] and the value of the deformation parameter β, and the total spin I. The values of the non–axiality parameter γ and the non-deformed oscillator parameter $\hbar\omega_0^0$, which are functions of the mass number A, the number of protons Z and the number of neutrons N [9] are also given in Table-1

It is seen from Table-1 that the calculated values of the electric quadrupole moments for the lithium nuclei ^{6}Li, ^{7}Li, and ^{8}Li are in good agreement with the corresponding experimental values for the case of the axially-symmetric shape, while the agreement with the experimental values for the other nuclei, ^{8}Be, ^{9}B, ^{10}B, ^{11}B, ^{2}C, and ^{14}N, is better in the case of the non-axially symmetric shape.

Table-1 Electric quadrupole moments of the nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}C, and ^{14}N

Nucleus	β	I^{π}	γ	$\hbar\omega_0^0$ (MeV)	Q_s (barns)	$Q_{exp.}$ (barns) [10]
^{6}Li	0.06 0.10	1^+ 1^+	0 10^o	--- 9.594	-0.00081 -0.00059	-.00083
^{7}Li	0.17 0.18	$\frac{3}{2}^-$ $\frac{3}{2}^-$	0 20^o	--- 11.796	-0.03992 -0.03978	-0.0408
^{8}Li	0.14 0.24	2^+ 2^+	0 20^o	--- 13.208	0.03121 0.03100	0.0317
^{9}Be	0.26 0.19	$\frac{3}{2}^-$ $\frac{3}{2}^-$	0 30^o	--- 12.561	0.03921 0.05214	0.0530
^{10}B	0.38 0.34	3^+ 3^+	0 30^o	--- 12.022	0.07403 0.08286	0.08472
^{11}B	0.37 0.41	$\frac{3}{2}^-$ $\frac{3}{2}^-$	0 30^o	--- 12.768	0.02762 0.03892	0.04085
^{12}C	0.18 0.13	2^+ 2^+	0 30^o	--- 12.238	0.06403 0.05921	0.0600
^{14}N	0.12 0.11	1^+ 1^+	0 10^o	--- 12.251	0.01898 0.01901	0.0193

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