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The Octupole Correlations In The ^{209}bi,^{209}pb Nuclei

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□ ABSTRACT □

The self consistent Hartree-Fock-Bogoliubov problem for nuclei with A=209 is solved by using the variation principle. Accordingly, the octupole coupling of $h_{\frac{9}{2}} \otimes d_{\frac{3}{2}}$ and $i_{\frac{13}{2}} \otimes h_{\frac{9}{2}}$ of the nucleus ^{209}Bi have been calculated for different octupole strengths. Moreover, the octupole coupling between $j_{\frac{15}{2}} \otimes g_{\frac{9}{2}}$ of the nucleus ^{209}Pb has been also calculated.

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ارتباطات ثماني القطب في النواتين ^{209}Bi, ^{209}Pb

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□ الملخّص □

حلت مسألة هارتري–فوك–بوغليوبوف للأنوية التي لها عدد كتلي A=209 باستخدام مبدأ التغير . وتم حساب ارتباط ثماني قطب للإنتقالين $h_{\frac{9}{2}}$ ® $d_{\frac{3}{2}}$ ، $i_{\frac{13}{2}}$ ® $h_{\frac{9}{2}}$ للنواة ^{209}Bi من أجل سعات مختلفة لاهتزاز ثماني القطب. وأخيراً حسبنا ارتباط ثماني قطب للإنتقال $j_{\frac{15}{2}}$ ® $g_{\frac{9}{2}}$ للنواة ^{209}Pb.

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1-INTRODUCTION:

The basic modes of excitations of nuclei are known in low-energy nuclear structure physics, namely single-particle and collective excitations. The later can be either vibrational motions (of spherical or deformed nuclei) or rotational motion (of prolate or oblate shaped ellipsoids). Since the different kinds of excitations lead to distinctively different patterns or sequences of excited levels we can obtain information on the nuclear structure of a specific isotope by measuring its excitation scheme. The structure depends on the interplay between protons and neutrons, and thus we finally test the strong nuclear force by comparing our model calculations with the experimental results.

Many properties of nuclei can be described in terms of a model of independent particles moving in an average potential whose space dependence closely follows the matter distribution. With unfilled shells, we find additional correlations between these particles. In the Bardeen, Cooper, and Schrieffer (BCS) model [1] we treat these correlations in a generalized single-particle picture by introducing quasi-particles and a new type of field, the pairing potential.

The Hartree-Fock-Bogoliubov (HFB) theory [1] generalizes and unifies both methods. Within this theory we look for the most general product wave functions consisting of independently moving quasi-particles.They are determined by a variational principle and take into account as many correlations as possible staying within a static single-particle picture. It turns out that within this approximation, the Hamiltonian reduces to two average potentials, the self-consistent field G, which are already known from the Hartree-Fock theory, and an additional pairing field D, known from the BCS theory. The field G contains all the long range particle hole (ph)-correlations which eventually lead to a deformed ground state (phase transition). On the other hand, Dsums up the short-range pairing correlations that can lead to a phase transition and a superfluid state.

The collective description of the octupole degree of freedom has been a long standing problem in nuclear physics[2]. The theoretical calculations predicted the existence of octupole stable deformations[3]. The features observed in nuclei are very similar to the once familiar from molecular physics. In molecules a stable octupole deformation leads to the appearance of rotational bands with the alternating parity levels connected by strong E1 intra-band transition[4].The nuclear structure community has devoted considerable theoretical and experimental effort to the study of the strong octupole correlation effects that are manifest only in the specific region of the periodic table. The octupole deformation can be understood through the single-particle level energy sequence for a harmonic-oscillator potential. In certain cases, an orbit is lowered into the next lowest major shell by the $\mathbf{l}^2$ and $\mathbf{l.s}$ terms, these intruder orbits can be strongly coupled by the octupole interaction. And the effect of octupole deformation on single particle levels is related to the octupole correlations[5]. There are some empirical indications that nuclei situated at certain regions can be even considered as reflection asymmetric in their ground states in agreement with a variety of model estimates.

The octupole correlation is quite strong and very important in heavy nuclei. In this paper we investigate the octupole deformation of the two nuclei ${}^{209}_{82}\mathbf{Pb}$, ${}^{209}_{83}\mathrm{Bi}$.

2-OCTUPOLE DEFORMATION:

It is well known that the surface of the nucleus can be expand into spherical harmonics[6]

$$R(\theta,\varphi) = R_0(1+\sum_{\lambda m} \alpha_{\lambda m}Y_{\lambda m}(\theta,\varphi)) \qquad (2.1)$$

λ=1 corresponds to dipole vibration, λ=2 corresponds to quadrupole vibration, λ=3 corresponds to an octupole vibration.

An octupole-deformed surface is given by

$$R(\theta,\varphi) = R_0(1+\sum_{m} \alpha_{3m}Y_{3m}(\theta,\varphi)) \qquad (2.2)$$

if we impose the axial symmetry (2.2) becomes

$$R(\theta,\varphi) = R_0(1+\alpha_{30}Y_{30}(\theta,\varphi)) \qquad (2.3).$$

For $\boldsymbol{\alpha}_{30} = 0.3$ and $\mathbf{R}_0 = 1$ we can see that this surface looks like a pear so that the octupole-deformed nuclei are often called pear-shaped nuclei.

To describe this case we use Hamiltonian[7]

$$\mathbf{H} = \mathbf{H}_s + \mathbf{H}_p + \mathbf{H}_{QQ} \qquad (2.4)$$

where $\mathbf{H}_s$ is the spherical single particle potential (Nilsson potential at zero deformation with corresponding single-particle energy E_k)

$$\mathbf{H}_s = \sum_k E_k(a_k^+ a_k + a_{\bar{k}}^+ a_{\bar{k}}) \qquad (2.5)$$

where k refer to the spherical harmonic oscillator state $|\mathbf{n,l,j,m}\rangle$, $\mathbf{a}_{\bar{\mathbf{k}}}^+ = \mathbf{T}\mathbf{a}_{\mathbf{k}}^+\mathbf{T}^{-1}$.

The Hamiltonian $\mathbf{H}_p$ are theoretical range pp-correlations[8]

$$\mathbf{H}_p = -\sum_{\tau \in \substack{\text{neutrons}\\ \text{protons}}} G_\tau \mathbf{P}_\tau^+ \mathbf{P}_\tau; \qquad (2.6)$$

where

$$\mathbf{P}_\tau^+ = \sum_{k\in\tau} a_k^+ a_{\bar{k}}^+ \qquad (2.7)$$

The last part are long-range ph-correlations defined by

$$\mathbf{H}_{QQ} = -\frac{1}{2}\sum_{\tau=0,1} K_3^{[\tau]}\mathbf{Q}_{30}^+[\tau]\mathbf{Q}_{30}[\tau] \qquad (2.8)$$

$$\mathbf{Q}_{30}[\tau = {}^{\text{proton}}_{\text{neutron}}] = \sum_{k,l\in\tau}(q_{kl}^{30}a_k^+a_l + q_{k\bar{l}}^{30}a_k^+a_{\bar{l}} + q_{\bar{k}\bar{l}}^{30}a_{\bar{k}}^+a_l + q_{\bar{k}\bar{l}}^{30}a_{\bar{k}}^+a_{\bar{l}}) \qquad (2.9)$$

in equation (2.9) q_{kl}^{30} are single-particle matrix elements in the spherical basis:

$$q_{kl}^{30} = \langle k|r^3Y_{30}|l\rangle \qquad (2.10)$$

Initially, only octupole deformation of the Y_{30} type where considered.

3-HARTREE-FOCK-BOGOLYUBOV SOLUTION

We assume that our vacuum is S-Symmetric (nuclei preserves the deformed average field with respect to reflections in planes perpendicular to the intrinsic axes 1 and 2) we obtain

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_{11} + \mathbf{H}_{20} + \mathbf{H}_4(AA) + \mathbf{H}_4(AB) + \mathbf{H}_4(BB) \qquad (3.1)$$

where

$$\mathbf{H}_0 = \sum_{ik} E_k (B^{*}_{\bar{k}} B^{i*}_{\bar{k}} + B^{\bar{i}}_{k} B^{\bar{i}*}_{\bar{k}}) - \sum_{t} G_t (\langle\ |\mathbf{P}_t|\ \rangle)^2 - \frac{1}{2}\sum_{t} K_3^{[t]} (\langle\ |\mathbf{F}_0^{(-)}[t]|\ \rangle)^2 \qquad (3.2)$$

and

$$\mathbf{H}_4(AB) = -\sum_{t} G_t (\mathbf{P}_t^{+}(A)\mathbf{P}_t(B) + \mathbf{P}_t^{+}(B)\mathbf{P}_t(A)) \qquad (3.3)$$

The self consistent HFB problem is solved by means of the Bogolyubov transformation form spherical particle operator a_i^{+}, a_i :

$$a_i = \sum_{k} (A^{i}_{k} a^{|}_{k} + B^{i}_{\bar{k}} a_{\bar{k}})$$

$$a_i^{+} = \sum_{k} (A^{\bar{i}}_{k} a^{|}_{k} + B^{\bar{i}}_{k} a_{k}) \qquad (3.4)$$

by substituting the equations (3.4) in the equation (3.1) we get the equation

$$\mathbf{H}_{HFB} = \mathbf{H}_0 + \mathbf{H}_{11} + \mathbf{H}_{20} \qquad (3.5)$$

and using variation principle [9]

$$\frac{d}{d|\ \rangle}\frac{\langle nljm|\mathbf{H}_{HFB}|nljm\rangle}{\langle nljm|nljm\rangle} = 0 \qquad (3.6)$$

The variation principle leads to the following system of equations for the amplitudes $A^{i}_{k}, B^{\bar{i}}_{k}, A^{i}_{\bar{k}}$ and $B^{i}_{\bar{k}}$ for the corresponding quasiparticle energy E_i [10]:

$$\begin{pmatrix} h^{(1)} & \tilde{\Delta} \\ \tilde{\Delta}^{+} & h^{(2)} \end{pmatrix}\begin{pmatrix} A^{i} \\ B^{i} \end{pmatrix} = E_i \begin{pmatrix} A^{i} \\ B^{i} \end{pmatrix} \qquad (3.7)$$

and

$$\begin{pmatrix} h^{(1)} & \tilde{\Delta} \\ \tilde{\Delta}^{+} & h^{(2)} \end{pmatrix}\begin{pmatrix} A^{\bar{i}} \\ B^{\bar{i}} \end{pmatrix} = -E_{\bar{i}} \begin{pmatrix} A^{\bar{i}} \\ B^{\bar{i}} \end{pmatrix} \qquad (3.8)$$

where[9]

$$\tilde{\Delta}^{+}_{kl} = \tilde{\Delta}_{kl} = -G_t \langle |P_t|\ \rangle d_{kl} = \Delta_t d_{kl}$$

$$h^{(2)} = -h^{(1)}$$

4-CALCULATIONS

The HFB variational principle leads to the following single-particle Hamiltonian $h^{(1)}$

$$h^{(1)} = \begin{pmatrix} -\frac{E}{2} & -x_3 f_0 K_3(m) \\ -x_3 f_0 K_3(m) & \frac{E}{2} \end{pmatrix} \qquad (4.1)$$

In equation(4.1) E is the energy difference between the spherical single-particle energies of two orbitals, x_3 is the octupole interaction strength, and

$$K_3(m) = \langle n_1 l_1 j_1 m | r^3 Y_{30}(\theta,\varphi) | n_2 l_2 j_2 m \rangle$$

$$= \langle n_1 l_1 | r^3 | n_2 l_2 \rangle \langle l_1 j_1 m | Y_{30}(\theta,\varphi) | l_2 j_2 m \rangle \qquad (4.2)$$

By applications of the Wigner-Echart theorem[1] to the term $\langle l_1 j_1 m | Y_{30}(\theta,\varphi) | l_2 j_2 m \rangle$ we obtain

$$\langle l_1 j_1 m | Y_{30}(\theta,\varphi) | l_2 j_2 m \rangle = (-1)^{j_1 - m} \begin{pmatrix} j_1 & 3 & j_2 \\ -m & 0 & m \end{pmatrix} \langle l_1 j_1 m \| Y_{30}(\theta,\varphi) \| l_2 j_2 m \rangle \qquad (4.3)$$

where

$$\langle l_1 j_1 m \| Y_{30}(\theta,\varphi) \| l_2 j_2 m \rangle = \left(\frac{1+(-1)^{l_1+l_2+3}}{2}\right) \sqrt{\frac{7(2j_1+1)(2j_2+1)}{4\pi}} (-1)^{j_1-\frac{1}{2}} \begin{pmatrix} j_1 & 3 & j_2 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \qquad (4.4)$$

In equation(4.3) and (4.4) $\begin{pmatrix} j_1 & 3 & j_2 \\ -m & 0 & m \end{pmatrix}$ are Wigner 3j symbols.

In equation(4.1) f_0 given by

$$f_0 = \sum_{i \le i_F} \langle n_i l_i j_i m | r^3 Y_{30}(\theta,\varphi) | n_i l_i j_i m \rangle \qquad (4.5)$$

and $\langle r^3 \rangle$ is given by

$$\langle r^3 \rangle = 4\pi \left[\frac{\Gamma(n+1)\Gamma(n'+1)}{\Gamma(n+\tau-\nu+1)\Gamma(n'+\tau-\nu'+1)} \right]^{\frac{1}{2}} \nu!\nu'!$$

$$\times \sum_{\sigma} \frac{\Gamma(\tau+\sigma+1)}{\sigma!(n-\sigma)!(n'-\sigma)!(\sigma+\nu'-n)!(\sigma+\nu-n)!} \qquad (4.6)$$

where

$$n = \frac{1}{2}(N-l), \quad n' = \frac{1}{2}(N'-l'), \quad \nu = \frac{1}{2}(l'-l+3), \quad \nu' = \frac{1}{2}(l-l'+3)$$

we use Nilsson model to calculate E the energy difrence between the spherical single-particle energies

5-RESULTS AND CONCLUSIONS

The only state observed in the nucleus with A=209 are single-particle and single hole states. The experimental study of the ^{209}Bi(d ^{3}He)^{208}Pb reaction found to populate the $I^{\pi} = 3^{-}$ state in the nucleus ^{208}Pb and that the process involve pickup of a $d_{\frac{3}{2}}$ proton[11].

In our calculations we have used values of the oscillator parameter, which depend on the mass number A, the number of neutrons N and the number of protons Z as follows [13]

$$\hbar\omega = \frac{38.6}{A^{\frac{1}{3}}[1 + \frac{1.646}{A} - 0.191\frac{(N-Z)}{A}]^{2}} \qquad (5.1)$$

In Table-1 we present the calculated values of the matrix elements of r^{3}, and K_3 for each transition of the two nuclei ^{209}Bi and ^{209}Pb.

The calculated values of the energy splitting are also given in this table.

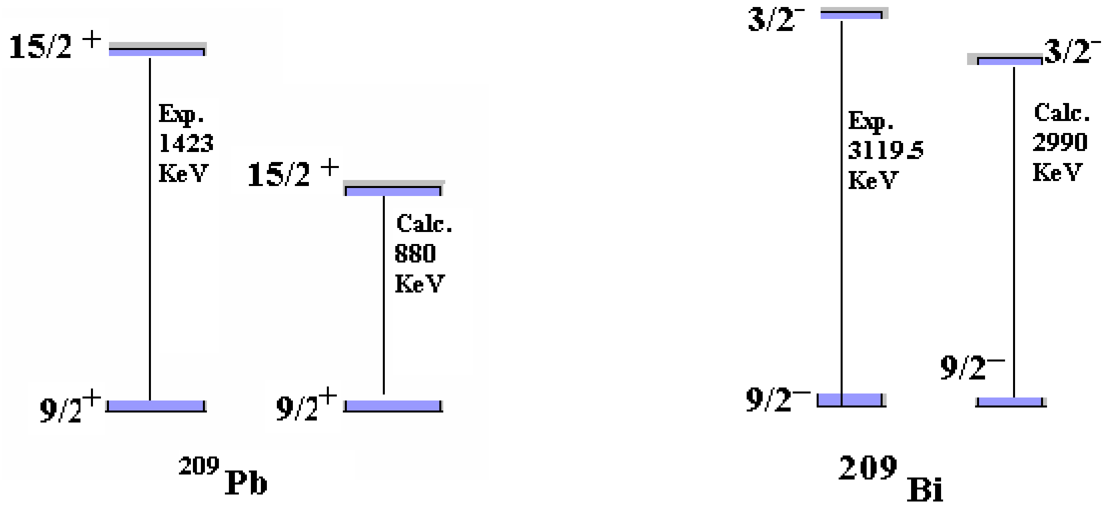
In Figures-1 we present the calculated and the corresponding energy splitting for the two nuclei ^{209}Bi and ^{209}Pb, respectively. In Figures-2 (a, b, and c) we present the variations of the octupole coupling with respect to the interaction strength for each nucleus.

The results of the calculations for octupole coupling of $h_{\frac{9}{2}} \otimes d_{\frac{3}{2}}$ and $i_{\frac{13}{2}} \otimes h_{\frac{9}{2}}$ of the ^{209}Bi nucleus have been in good agreement with the experiment for octupole strength 1.017 MeV fm^{-6}, the addition of an octupole quantum $i_{\frac{13}{2}} \otimes h_{\frac{9}{2}}$ to the ground state of ^{209}Bi is expected to give rise to a septuplet of states [12] $(h_{\frac{9}{2}}\ 3^{-})I$ with $I = \frac{3}{2}, \frac{5}{2}, \ldots\ldots, \frac{15}{2}$.

The observed small splitting of the multiplet components implies a weak coupling between odd proton and the octupole quantum, here it is only 15% for octupole strength 1.017.

For ^{209}Pb, the coupling matrix element between the $j_{\frac{15}{2}}$ and $g_{\frac{9}{2}}$ configuration has the value -0.88MeV at the strength 0.02.

Nucleus	Transition	$\langle n_1 l_1 \| r^{3} \| n_2 l_2 \rangle$ fm^{3}	$k_3(m)$	ΔE(keV)
^{209}Pb	$j_{\frac{15}{2}} \otimes g_{\frac{9}{2}}$	253.5	-10.71	58
^{209}Bi	$h_{\frac{9}{2}} \otimes d_{\frac{3}{2}}$	341.2	2.96	69
^{209}Bi	$i_{\frac{13}{2}} \otimes h_{\frac{9}{2}}$	233.5	-0.44	151



$c = 0.02\,\mathrm{MeVfm}^{-6}$ $\quad$ $c = 1.017\,\mathrm{MeVfm}^{-6}$

Fig-1

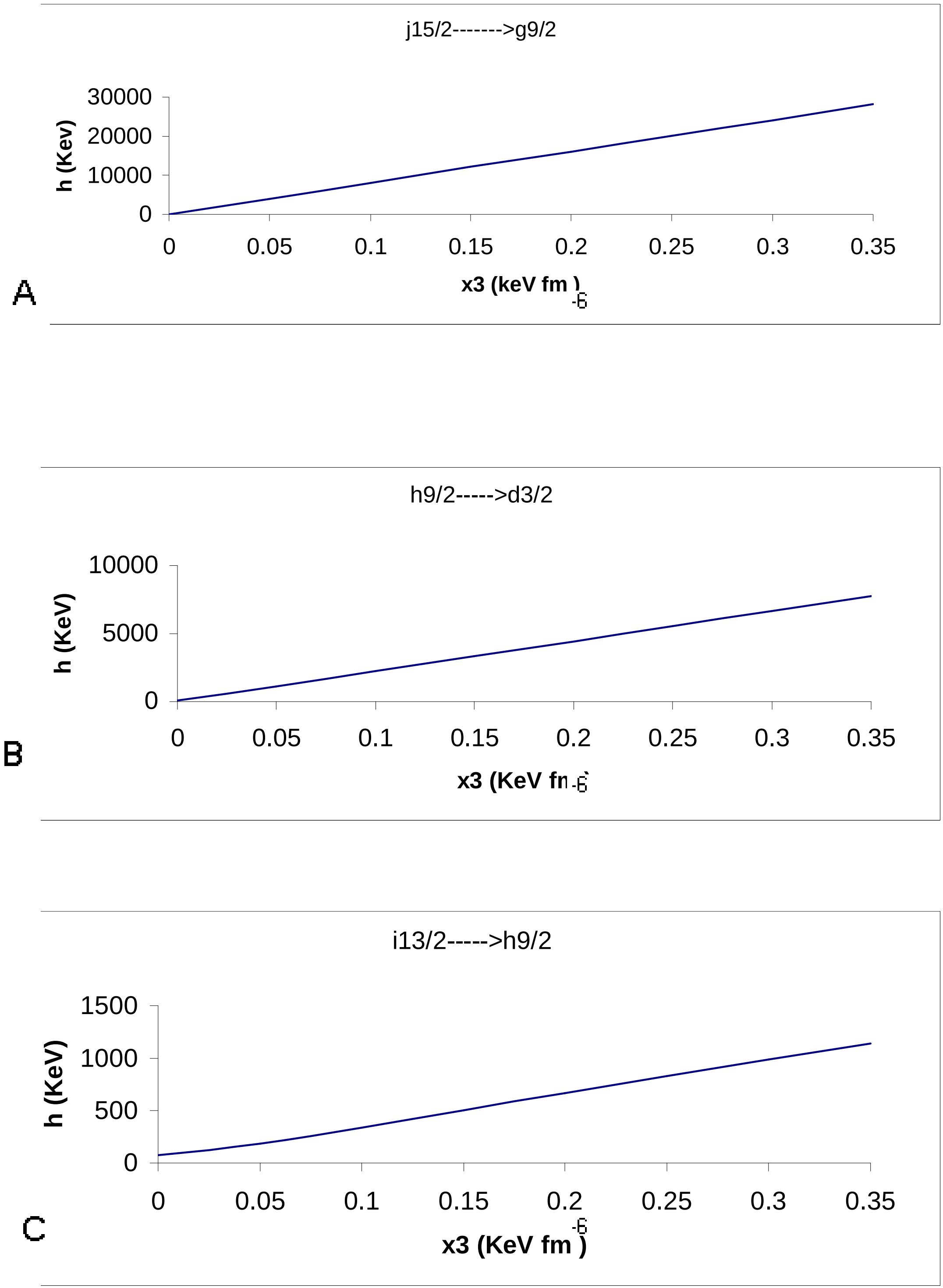


fig (2)pointed out that the energy splitting h is very sensitive to the chosen octupole strength.

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