مجلة جامعة تشرين للبحوث والدراسات العلمية _ سلسلة العلوم الأساسية المجلد (30) العدد (1) 2008
Tishreen University Journal for Research and Scientific Studies - Basic Sciences Series Vol. (30) No. (1) 2008

The Electron Spin Derivation and the *Zitterbewegung* in the Dirac Equation

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(Received 9 / 10 / 2007. Accepted 28/1/2008)

☐ ABSTRACT ☐

The integration between the special relativity theory and quantum mechanics through the Dirac equation yielded many paradoxes that remained unsolved until the last years, like the *Zitterbewegung* problem. Besides, the spin prediction from the Dirac equation could be identified only with non-relativistic approximations (Pauli and Foldy-Wouthysen).

In this paper, we showed that the derivation of the spin and its magnetic moment can be done through a classical treatment without approximation. By this approach a modified Dirac equation was obtained, which also eliminates the problem of the *Zitterbewegung*.

Key words:
Dirac equation, Eectron spin, *Zitterbewegung*.

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مجلة جامعة تشرين للبحوث والدراسات العلمية – سلسلة العلوم الأساسية المجلد (30) العدد (1) 2008
Tishreen University Journal for Research and Scientific Studies - Basic Sciences Series Vol. (30) No. (1) 2008

إشتقاق سبين الإلكترون والحركة الارتجاجية في معادلة ديراك

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(تاريخ الإيداع 9 / 10 / 2007. قُبِل للنشر في 2008/1/28)

□ الملخّص □

أدى الجمع بين نظرية النسبية الخاصة و ميكانيك الكم من خلال معادلة ديراك (Dirac) إلى نشوء عدة مسائل، والتي بقيت دون حل حتى السنوات الأخيرة. و منها مشكلة الحركة الارتجاجية (*Zitterbewegung*)، بالإضافة إلى أن إظهار سبين الإلكترون و عزمه المغاطيسي من معادلة Dirac يمكن أن يتم فقط بشكل غير مباشر و ذلك باستخدام عمليات تقريب لانسبوية كتقريبي Pauli و Foldy-Wouthysen.

في هذا البحث نبين أنه يمكن استنتاج سبين الإلكترون و عزمه المغناطيسي من خلال معالجة تقليدية دون استخدام عمليات تقريب، و ذلك بتعديل معادلة ديراك (Dirac equation)، و الذي يؤدي أيضا لحل مشكلة الحركة الارتجاجية.

كلمات مفتاحية:
معادلة Dirac ، سبين الإلكترون، الحركة الارتجاجية.

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Introduction:

The integration of the special relativity theory with quantum mechanics has yielded many paradoxes that remained unsolved until recent years. A short time after the publication of the Dirac equation, a serious problem was discovered by Schrödinger [1], this problem was known as *Zitterbewegung*. Similarly, it was impossible to write directly a non-relativistic equation for spin-1/2 particles, and that it could only be derived as a non-relativistic limit of the relativistic Dirac equation. Therefore, the Pauli equation for the theory of spin was derived as a non-relativistic limit of the relativistic Dirac equation, even though, that was known in standard quantum mechanics as a direct proof of the fundamentally relativistic nature of the spin [2]. However, in 1984 this supposition was questioned by W. Greiner [3] when he derived the spin from the non-relativistic quantum mechanics, i.e. he derived the spin from the Schrödinger equation. A prominent attempt to eliminate the problem of *Zitterbewegung* was by E. G. Bakhoum [4], who requires a modification in the mass-energy equivalence principle. Bakhoum introduced a new total relativistic energy formula $E = mv^2$ instead of Einstein's $E = mc^2$, where m is the relativistic mass of the particle, and v is the particle velocity.

This paper carries Bakhoum's work a step further as we have derived Einstein's equation $E = mc^2$ without using the special relativity theory. Instead, we started from the classical physics laws like the Lorentz force law and Newton's second law [5,6,7,8,9]. The energy formula of a particle $E = mv^2$ allows reconciliation between the de Broglie wave theory and the framework of the relativistic physics without the usual contradictions. In this paper, by starting from the new total relativistic energy formula $E = mv^2$, we derived a modified Dirac equation and we obtained the same result of Bakhoum concerning the *Zitterbewegung*. Furthermore, we revealed that the spin of the electron and its magnetic moment can be derived without using approximations.

Aim and Importance:

The importance of this paper is that it aims to solve an important problem concerning the essential characteristic of the electron, which is the electron spin. The electron spin plays a fundamental role in many physical effects in several branches of physics, such as solid state physics and atomic physics and the physics of compact stars. Moreover, many modern technological applications in the fields of electronics, nanotechnology and quantum computation depend on spin.

Methodology:

We use in our calculations and derivation of equations mathematical methods and symbols that are familiar internationally in standard textbooks in this specialization of physics.

This research had been done at Department of Physics in the Faculty of Sciences at both the University of Aleppo and Tishreen University, through the years 2006-2007.

The Relativistic Dirac Equation:

The early twentieth century saw two major revolutions in the way physicists understand the world. The first one was quantum mechanics, and the other was the theory of relativity. Important results also emerged when these two ideas were brought together, and one of these results is the spin of the electron that was known as relativistic effect.

When calculating kinetic energy relativistically using Lorentz transformation instead of Newtonian mechanics, Einstein discovered that the amount of energy is directly proportional to the mass of a particle

$$E = mc^2. \qquad (1)$$

The energy and momentum of a particle are then related by the principal equation governing the dynamics of a free particle

$$E^2 = c^2\boldsymbol{p}^2 + m_0^2c^4. \qquad (2)$$

Following Dirac, by taking into account time dependence like in Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi. \qquad (3)$$

As in the non-relativistic case and assuming that the energy operator $\hat{H}$ can be expressed in terms of the momentum operator $\hat{\boldsymbol{p}}$ in the same way as E is related to p in the classical limit.

Hence, using Eq. (2) and (3) we obtain

$$i\hbar\frac{\partial}{\partial t}\psi = \sqrt{c^2\hat{\boldsymbol{p}}^2 + m_0^2c^4}\,\psi.$$

One of the conditions imposed by Dirac in writing down a relativistic equation for the electron was that the ‘‘square’’ of that equation will give the Klein – Gordon equation. Imposing the additional condition of linearity of $\hat{H}_D$ in the components $\hat{p}_k$ of the momentum, [10,11] led Dirac to Eqs. (4) and (5)

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}_D\psi, \qquad (4)$$

where

$$\hat{H}_D = c(\boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}}) + m_0c^2\beta, \qquad (5)$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad k = 1,2,3, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad (6)$$

where σ_k are the two by two Pauli matrices, and I is the two by two identity matrixes.

The Pauli Equation as a Non-relativistic Limit of the Dirac Equation:

In standard quantum mechanics, it is believed that it is not possible to directly extend the Schrödinger equation to describe spinors. So, the Pauli equation must be derived from the Dirac equation by taking its non-relativistic limit. This is in particular the case for the Pauli equation which predicts the existence of an intrinsic magnetic moment for the electron and gives its correct value only when it is obtained as the non-relativistic limit of the Dirac equation.

The Dirac equation for a relativistic electron moving in a constant magnetic field could be written as following

$$i\hbar\frac{\partial}{\partial t}\psi = [c\boldsymbol{\alpha}\cdot(\hat{\boldsymbol{p}} - \frac{e}{c}\boldsymbol{A}) + \beta m_0c^2]\psi. \qquad (7)$$

We can now derive the Pauli equation following the standard method of eliminating small components in the Dirac spinors. We consider a two-component representation, where the four-component spinor ψ is decomposed into two two-component spinors ψ_b and ψ_s

$$\psi = \begin{pmatrix} \psi_b \\ \psi_s \end{pmatrix}, \quad \text{و} \quad \psi_b = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \ \psi_s = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}. \qquad (8)$$

In the non-relativistic limit, the rest energy m_0c^2 becomes dominant. Therefore, the two component solution is approximately

$$\psi_{b,s} = e^{-\frac{im_0c^2t}{\hbar}} \psi^0_{b,s} . \qquad (9)$$

Substituting this non-relativistic solution into the Dirac equation, equation (7), in the Dirac representation gives

$$i\hbar \frac{\partial}{\partial t}\psi_b^0 = c\boldsymbol{\sigma}\cdot(i\hbar\nabla - \frac{e}{c}\boldsymbol{A})\psi_s^0 \ . \qquad (10a)$$

$$i\hbar \frac{\partial}{\partial t}\psi_s^0 = c\boldsymbol{\sigma}\cdot(i\hbar\nabla - \frac{e}{c}\boldsymbol{A})\psi_b^0 - 2m_0c^2\psi_s^0 \ . \qquad (10b)$$

When the kinetic energy is small compared to the rest energy, then ψ_s^0 will slowly vary as a function of time, so

$$\left| i\hbar \frac{\partial}{\partial t}\psi_s^0 \right| \ll \left| m_0c^2\psi_s^0 \right| . \qquad (11)$$

With this approximation, equation (10b) becomes

$$0 = c\boldsymbol{\sigma}\cdot(i\hbar\nabla - \frac{e}{c}\boldsymbol{A})\psi_b^0 - 2m_0c^2\psi_s^0 .$$

This gives

$$\psi_s^0 = \frac{\boldsymbol{\sigma}\cdot(i\hbar\nabla - \frac{e}{c}\boldsymbol{A})}{2m_0c}\psi_b^0 . \qquad (12)$$

The lower component ψ_s^0 is generally referred to as the 'small' component of the wave function ψ, relative to the 'large' component ψ_b^0.

Substituting the expression ψ_s^0, given by equation (12), into equation (10a), we obtain

$$i\hbar \frac{\partial}{\partial t}\psi_b^0 = \frac{1}{2m_0}\left(\boldsymbol{\sigma}\cdot(i\hbar\nabla - \frac{e}{c}\boldsymbol{A})\right)\left(\boldsymbol{\sigma}\cdot(i\hbar\nabla - \frac{e}{c}\boldsymbol{A})\right)\psi_b^0 .$$

Finally, by using the well-known identity

$$(\boldsymbol{\sigma}\cdot\boldsymbol{a})(\boldsymbol{\sigma}\cdot\boldsymbol{b}) = \boldsymbol{a}\cdot\boldsymbol{b} + i\boldsymbol{\sigma}\cdot(\boldsymbol{a}\times\boldsymbol{b}) .$$

We obtain, $\boldsymbol{B} = \nabla\times\boldsymbol{A}$, being the magnetic field

$$i\hbar \frac{\partial}{\partial t}\psi_b^0 = [\frac{1}{2m_0}(i\hbar\nabla - \frac{e}{c}\boldsymbol{A}) - \frac{e\hbar}{2m_0c}\boldsymbol{\sigma}\boldsymbol{B}]\psi_b^0 \ . \qquad (13)$$

This is the well known Pauli equation. So, the Pauli equation in the theory of spin was derived as a non-relativistic limit of the relativistic Dirac equation, and it was considered in standard quantum mechanics as a direct proof of the fundamentally relativistic nature of the spin.

Derivation of Modified Dirac Equation and Its Solutions:

In several recent papers we suggested another way to account for the Lorentz Transformation and its kinematical effects in relativistic electrodynamics as well as in relativistic mechanics. And by following the same approach we derived Einstein's equation $E = mc^2$ from classical physical law such as the Lorentz force law.

As demonstrated in the papers [6,7] the relativistic transformation relations and relativistic formulae are produced without the Lorentz Transformation and its kinematical effects. We obtained the known relativistic mass and energy formulae

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}, \text{ and } E = mc^2. \qquad (14)$$

It is simple to prove that Eqs.(14) leads to the following equation

$$m^2c^4 = m^2c^2v^2 + m_0^2c^4. \qquad (15)$$

If we multiply Eq.(15) by v^2/c^2 and rearrange, we get

$$m^2v^4 = c^2\mathbf{p}^2 - m_0^2c^2v^2.$$

Now if we let $H = mv^2$ in the last equation, then we have

$$H^2 = c^2\mathbf{p}^2 - m_0^2c^2v^2. \qquad (16)$$

Multiplying and at the same time dividing the last term in Eq.(16) by m^2, we obtain

$$H^2 = c^2\mathbf{p}^2\left(1-\frac{m_0^2}{m^2}\right) = c^2\mathbf{p}^2\left[1-\left(1-\frac{v^2}{c^2}\right)\right]. \qquad (17)$$

So, we finally get

$$H^2 = v^2\mathbf{p}^2. \qquad (18)$$

Following the method of Dirac we can write the new Hamiltonian as

$$H = v\,\alpha_j p_j, \qquad (19)$$

where the matrices α_j must satisfy the following condition

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}, \qquad (20)$$

which is satisfied also by the Dirac matrices; so the matrices α_j are considered the Dirac matrices themselves.

Replacing H with the operator $i\hbar\frac{\partial}{\partial t}$ and p with the operator $-i\hbar\nabla$ in Eq.(15), then the modified Dirac equation for a free electron can be written as following

$$\frac{\partial}{\partial t}\psi + v\,\alpha_j\nabla_j\psi = 0. \qquad (21)$$

We will continue to find the Eigen functions of the new Hamiltonian for free electron to show that there is no contradiction between our results and Dirac's basic conceptions. Solutions to Eq.(21) are plane waves which can be written in the following form

$$\Psi(x,t) = N\begin{pmatrix}\varphi(x,t)\\ \chi(x,t)\end{pmatrix} = N\begin{pmatrix}\varphi(x)\\ \chi(x)\end{pmatrix}e^{\frac{-iE_v t}{\hbar}}. \qquad (22)$$

where N is the normalization constant, and $\begin{pmatrix}\varphi(x)\\ \chi(x)\end{pmatrix}$ is a four component spinor.

Substituting Eq.(22) in Eq.(21), and considering that

$$\begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix} = \begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} e^{\frac{ipx}{\hbar}},$$

where $\varphi_0 = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $\chi_0 = \begin{pmatrix} u_3 \\ u_4 \end{pmatrix}$ are two-component spinors, we find

$$E_v \begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} = v\, p_j \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix}. \qquad (23)$$

From Eq.(23) we obtain these two equations

$$\left.\begin{aligned} E_v \varphi_0 - v p_j \sigma_j \chi_0 = 0 \\ E_v \chi_0 - v p_j \sigma_j \varphi_0 = 0 \end{aligned}\right\}. \qquad (24)$$

These two equations have solution if

$$\begin{vmatrix} E_v & -v p_j \sigma_j \\ -v p_j \sigma_j & E_v \end{vmatrix} = 0,$$

or equivalently

$$E_v^{\,2} = v^2 \mathbf{p}^2. \qquad (25)$$

From Eqs.(24) and normalization condition we find

$$\chi_0 = \frac{v\, p_j \sigma_j}{E_v} \varphi_0 \quad \text{and} \quad \varphi_0 = \frac{v\, p_j \sigma_j}{E_v} \chi_0. \qquad (26)$$

From these two equations we obtain

$$\chi_0 = \frac{(v\, p_j\, \sigma_j)^2}{E_v^{\,2}} \chi_0. \qquad (27)$$

Using Eq.(25) in Eq.(27), we find that χ_0 can take these two representations

$$\chi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \chi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

From the above calculations we find

$$\Psi(x,t) = N \begin{pmatrix} \varphi(x,t) \\ \dfrac{v\, p_j\, \sigma_j}{E_v} \varphi(x,t) \end{pmatrix} = N \begin{pmatrix} \varphi_0 \\ \dfrac{v\, p_j\, \sigma_j}{E_v} \varphi_0 \end{pmatrix} e^{\frac{i}{\hbar}(px - E_v t)}. \qquad (28)$$

By calculating the normalization constant N for positive and negative energy solutions, we find

$$N = \frac{|E_v|}{\sqrt{E_v^{\,2} + (vp)^2}}. \qquad (29)$$

So, the two solutions of the modified Dirac equation that corresponds to the positive energy $E = +\sqrt{\mathrm{v}^2 \mathbf{p}^2}$ with two different stats of the electron spin orientation are

$$\psi_{+}^{\uparrow} = \frac{|E|}{\sqrt{E^2 + (\mathrm{v}\mathbf{p})^2}} \begin{pmatrix} 1 \\ 0 \\ \frac{\mathrm{v}p_3}{E} \\ \frac{\mathrm{v}(p_1 + ip_2)}{E} \end{pmatrix} e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar} , \qquad (30a)$$

$$\psi_{+}^{\downarrow} = \frac{|E|}{\sqrt{E^2 + (\mathrm{v}\mathbf{p})^2}} \begin{pmatrix} 0 \\ 1 \\ \frac{\mathrm{v}(p_1 - ip_2)}{E} \\ \frac{-\mathrm{v}p_3}{E} \end{pmatrix} e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar} . \qquad (30b)$$

Similarly, the two solutions of the modified Dirac equation that corresponds to the negative energy $E = -\sqrt{\mathrm{v}^2\mathbf{p}^2}$ with two different stats of the electron spin orientation are

$$\psi_{-}^{\uparrow} = \frac{|E|}{\sqrt{E^2 + (\mathrm{v}\mathbf{p})^2}} \begin{pmatrix} \frac{\mathrm{v}p_3}{E} \\ \frac{\mathrm{v}(p_1 + ip_2)}{E} \\ 1 \\ 0 \end{pmatrix} e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar} , \qquad (31a)$$

$$\psi_{-}^{\downarrow} = \frac{|E|}{\sqrt{E^2 + (\mathrm{v}\mathbf{p})^2}} \begin{pmatrix} \frac{\mathrm{v}(p_1 - ip_2)}{E} \\ \frac{-\mathrm{v}p_3}{E} \\ 0 \\ 1 \end{pmatrix} e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar} . \qquad (31b)$$

Although we started from classical electrodynamics, the analysis in our paper is still entirely relativistic by the formula $E_v = mv^2$ instead of $E = mc^2$. We will also show now that the modified Hamiltonian, Eq.(19), leads to the same result of Bakhoum concerning the Zitterbewegung. Furthermore, we reveal that the spin of the electron and its magnetic moment could be derived without using approximation.

Calculating the Velocity from the Modified Dirac Hamiltonian:

In the Dirac theory for spin 1/2 particle, the velocity operator is $\hat{v} = c\hat{\alpha}$. The problem of the *Zitterbewegung* in Dirac's Hamiltonian corresponds to the first term in the Hamiltonian i.e. the term $c\,\boldsymbol{\alpha}\cdot\mathbf{p}$. Actually, the particle velocity component on the ox axis when calculated from the Poisson bracket using the Dirac Hamiltonian gives the following result [4]

$$\dot{x} = [x, H] = \frac{\partial H}{\partial p_x} = c\alpha_1 . \qquad (32)$$

From this equation, it is clear that $\dot{x}$ will be always $\pm c$, with c being the speed of light in vacuum. So, the operator $\hat{v} = c\hat{\alpha}$ is inadequate in two aspects. The first one is that its eigenvalues are $+c$ and $-c$, a result which is in obvious contradiction with the physical reality, the other one is that it is not proportional to the linear momentum. To overcome these shortcomings, E.G. Bakhoum obtained a Hamiltonian that is written as following [4].

$$H = \pm v \sum_r P_r \beta_r \quad ,$$

Where β_r are matrices that satisfy these two conditions

$$\beta_r^{\ 2} = I \ , \quad \text{and} \quad \beta_j \beta_k + \beta_k \beta_j = 0.$$

Unlike the Dirac result for the eigenvalue of the particle velocity, Bakhoum's result is in agreement with the experimental observation, since

$$\dot{x} = [x, H] = \frac{\partial H}{\partial P_1} = v\,\beta_1 .$$

From this equation, it is clear that $\dot{x}$ will be $+v$ or $-v$.

Another attempt that dealt with the problem of the *Zitterbewegung* was by Recami *et al.* [12,13,14]. They alleged that the *Zitterbewegung* is necessary for the quantum phenomenon of spin, and gave it a physical (classical) meaning. In this paper it is shown that by the modified Dirac Hamiltonian the velocity operator really retrieves the classical relation between velocity and momentum. So, we will see how we get the eigenvalue of the particle velocity from Eq.(25), with the problem of the *Zitterbewegung* canceled.

Multiplying Eq.(21) from the left with ψ^+, we obtain

$$\psi^+ \tfrac{\partial}{\partial t} \psi + v\, \psi^+ \hat{\alpha}_j \nabla_j \psi = 0 \cdot \qquad (33)$$

Taking the hermitean conjugate of Eq.(21), we get

$$\tfrac{\partial}{\partial t} \psi^+ + v\, \nabla_j \psi^+ \hat{\alpha}_j = 0 \cdot \qquad (34)$$

Multiplying the last equation from the right with ψ, we have

$$\tfrac{\partial}{\partial t} \psi^+ \psi + v\, \nabla_j \psi^+ \hat{\alpha}_j \psi = 0 \cdot \qquad (35)$$

Summing the tow equations (33) and (35), we get

$$\tfrac{\partial}{\partial t} (\psi^+ \psi) + \nabla_j (\psi^+ v\, \hat{\alpha}_j \psi) = 0 \cdot \qquad (36)$$

Comparing this equation with the familiar continuity equation

$$\tfrac{\partial}{\partial t} \rho + \nabla \cdot \mathbf{J} = 0, \qquad (37)$$

we identify that

$$\mathbf{J} = v\,\psi^+ \boldsymbol{\alpha} \psi . \qquad (38)$$

According to the known formula $\mathbf{J} = \rho \mathbf{v}$, we deduce that the expected value of the particle velocity obtained here from the modified Dirac equation equals $\pm v$.

The Dirac Hamiltonian *Zitterbewegung* referred to in some literatures as a result from interference between two positive and two negative energy components of the Dirac spinors. The modified Dirac Hamiltonian *Zitterbewegung* might be considered also resulting from interference between two positive and two negative energy components. It was shown that the expected value of the particle velocity obtained here from the modified Dirac equation always equal the velocity that the particle moves with, as observed in the laboratory the force-free electron can move at any velocity less than *c*. The difference between the interpretation of the "Zitterbewegung" from our modified Dirac Hamiltonian and from the Dirac Hamiltonian is now to be expected.

Derivation of Pauli Equation without Approximation Methods:

The most important result of the Dirac equation was presenting a theoretical description of the spin of the electron and its magnetic moment, that did not appear directly from the Dirac equation but using approximations (Pauli, Foldy-Wouthysen). This is not the case when we derive the spin of the electron and its magnetic moment from the new Hamiltonian, Eq.(19).

As we know, the momentum is replaced in the following way to include the effects of the magnetic field in the Dirac equation

$$\mathbf{p} \rightarrow \mathbf{p} - \frac{e}{c}\mathbf{A} . \qquad (39)$$

We apply the same idea to the new Hamiltonian in Eq.(24) which can be written as

$$H = v[\boldsymbol{\alpha} \cdot (\mathbf{p} - \frac{e}{c}\mathbf{A})] . \qquad (40)$$

By following the method of Bakhoum in his paper [4], i.e. by squaring Eq.(40), then we have a scalar equation and dividing it on mv^2 we get

$$H = \frac{1}{m}[\boldsymbol{\alpha} \cdot (\mathbf{p} - \frac{e}{c}\mathbf{A})]^2 . \qquad (41)$$

In the reference [15] the following equation holds

$$[\boldsymbol{\alpha} \cdot (\mathbf{p} - \frac{e}{c}\mathbf{A})]^2 = (\mathbf{p} - \frac{e}{c}\mathbf{A})^2 - \frac{e\hbar}{c}\,\boldsymbol{\sigma}_4 \cdot \mathbf{B} , \qquad (42)$$

where the matrices $\boldsymbol{\sigma}_4$ in Eq.(42) are the 4×4 Pauli matrices that have the following representation

$$\boldsymbol{\sigma}_4 = \begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}$$

Substituting Eq.(42) in Eq.(41), we finally obtain

$$H = \frac{1}{m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 - \frac{e\hbar}{mc}\boldsymbol{\sigma}_4 \cdot \mathbf{B} \qquad (43)$$

The second term on the r.h.s. of Eq.(43) represents the interaction of the electron spin magnetic moment with the magnetic field. An important characteristic of Eq.(43) is that we did not use any kind of approximation to reach it. Therefore, we got the relativistic mass m not the rest mass m_0, also the Pauli matrices here $\boldsymbol{\sigma}_4$ are 4×4 matrices. That is why Eq.(43) could be regarded as a relativistic Pauli equation.

The usual Pauli equation can be obtained from Eq.(43) by canceling terms containing $\frac{v}{c}$, since for $m \approx m_0$ we have $H = m_0 v^2$, so we get

$$\frac{1}{2}m_0 v^2 = \frac{1}{2m_0}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 - \frac{e\hbar}{2m_0 c}\boldsymbol{\sigma}_4 \cdot \mathbf{B} . \qquad (44)$$

Equation (44) is the usual Pauli equation, and without the presence of the magnetic field the Equation (44) is reduced to

$$\frac{1}{2m_0}\boldsymbol{p}^2 = \frac{1}{2}m_0\boldsymbol{v}^2 .$$

That means the electron has a spin magnetic moment $\boldsymbol{\mu} = -\frac{e\hbar}{2m_0 c}\boldsymbol{\sigma}$, and the magnetic moment interacts with an external magnetic field. So the corresponding contribution to the energy is $-\boldsymbol{\mu}\cdot\mathbf{B}$. We recognize here the classical Pauli equation for the theory of spin with the correct gyromagnetic factor $g = 2$ for free electron.

Conclusion:

There exists an inconsistency between Einstein's special relativity and the De Broglie wave mechanics, and it has never been resolved from the viewpoint of relativistic physics for a long time [8,9]. A more suitable method to deal with this contradiction is to develop the applicability of the classical physics laws to all particle velocities i.e. to expand the appropriateness of these laws to deal with the relativistic domain. Following this approach, a modified Dirac equation can be derived using classical description as it is shown in this paper.

Bakhoum showed how the modern physics as we know can be understood on the basis of the equation $H = mv^2$. In particular, Einstein's equation $H = mc^2$ becomes a special case of the broader equation $H = mv^2$. The Hamdan *et al.* work carries Bakhoum's work a step further, since recently we derived the formula $H = mv^2$ without using the special relativity theory, but starting from the Lorentz force law and the relativity principle. In this paper, we derived modified Dirac equation, and we obtained the same result of Bakhoum concerning the *Zitterbewegung*. Further, we got an additional advantage, where the modified Dirac equation, Eq.(21), directly leads to relativistic Pauli equation without using approximation methods. Unlike the Dirac equation and its predications, our modified Dirac equation and its results remove the conceptual difficulties with the problems of the *Zitterbewegung* and approximation methods. Up till now, the thing was not expected to happen. So our results are new and no such results exist in the scientific literature concerning the subject of this paper.

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