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Improved Formulation of the Optimum Design of a Mono-symmetrical I Section of Cold Formed Thin Walled Beams

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□ ABSTRACT □

In this paper we one of the structural optimization problems is discussed. It concerns the case of a thin walled beam made by a cold forming procedure. Hereafter, the paper presents an improved formulation of the geometric and technological constraints used in the optimum design problem of a mono-symmetrical I section under a pure bending loading. After formulating this problem properly, the paper proposes an efficient genetic algorithm; then, it shows how to implement it using a MATLAB program. At the end, the effectiveness of this adapted genetic algorithm program is examined by comparing its results with the solution given by the graphical method in a special case.

Key words: optimum structural design, thin walled beams, genetic algorithm

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صياغة محسنة لمسألة التصميم الأمثل لمقطع I نظامي في جائز رقيق الجدران وملفوف على البارد تناظرياً

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□ ملخّص □

نناقش في هذه المقالة في إحدى مسائل الأمثلة الإنشائية. تعنى هذه المسألة بجائز مقطعه رقيق الجدران. وفيما يلي نقدم صياغة محسنة لقيود التصميم الهندسية (الجيومترية) والتقنية المستخدمة في تصميم مقطع I نظامي وملفوف تناظرياً من صفائح رقيقة تحت تأثير حالة تحميل بعزمين طرفيين. بعد عرض هذه الصياغة، سنقدم خوارزمية جينية ونبين برمجتها باستخدام بيئة ماتلاب. وفي النهاية سنتفحص فعالية هذه الخوارزمية بمقارنة نتائجها مع نتائج الحل التخطيطي الممكن في حالة خاصة.

الكلمات المفتاحية: التصميم الإنشائي الأمثل، الجيزان رقيقة الجدران، الخوارزميات الجينية.

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Introduction:

Cold-formed thin-walled beams with open cross sections are very commonly used in both temporary and permanent civil engineering constructions. These beams are used in practice as primary or secondary load bearing elements. The technology of cold rolling steel and aluminum structural member fabrication is now very extended. Its market puts the design engineer in front of a great variety of functionally or structurally available shape of the open cross section. In such a case, one of the best way to choose the section, i.e. determine its parameters, is to use the techniques of structural optimization. The best way to boost the use of these techniques by the structural engineers is to incorporate them in the commercial structural design and analysis software. This incorporation needs the elaboration of new simple and general algorithms of structural optimization. In this paper we have chosen to develop one of these algorithms and to examine its limitations. Our choice of the genetic algorithm method is justified by its mathematical simplicity and by its generality.

Importance and aims of the paper:

Scientific literature has witnessed an increasing number of publications [1] showing the results of applying the techniques of structural optimization in the design of open cross sections. This type of structures becomes more and more the best solution for many applications in the domain of civil, mechanical and aerospace engineering.

A deep surveying of this literature showed that we can improve the formulation given by some authors [2] & [3], from where we can fix the first aim of this study as to make this formulation more consistent with the real situation faced by design engineers and more adapted with the issue of availability of these sections in the industrial market. Our new formulation in this paper concerns the mono-symmetrical I section cold formed thin walled beam see Fig. 1.

The second aim of this paper is to expose an enhanced mathematical procedure of the optimization of the section shape, based on an adapted version of a MATLAB Genetic Algorithm program [4] & [5], which minimizes the area of the cross section while respecting the general strength, and local and global buckling constraints. More than this, we will examine the validity of this algorithm, by two ways, first, by studying the results of its application to a simple benchmark problem, i.e. the case of a simple beam in a pure bending. The second way to examine this algorithm, is to compare its analytical results with the results of a its simplified version of two variables, where the solution is graphically possible.

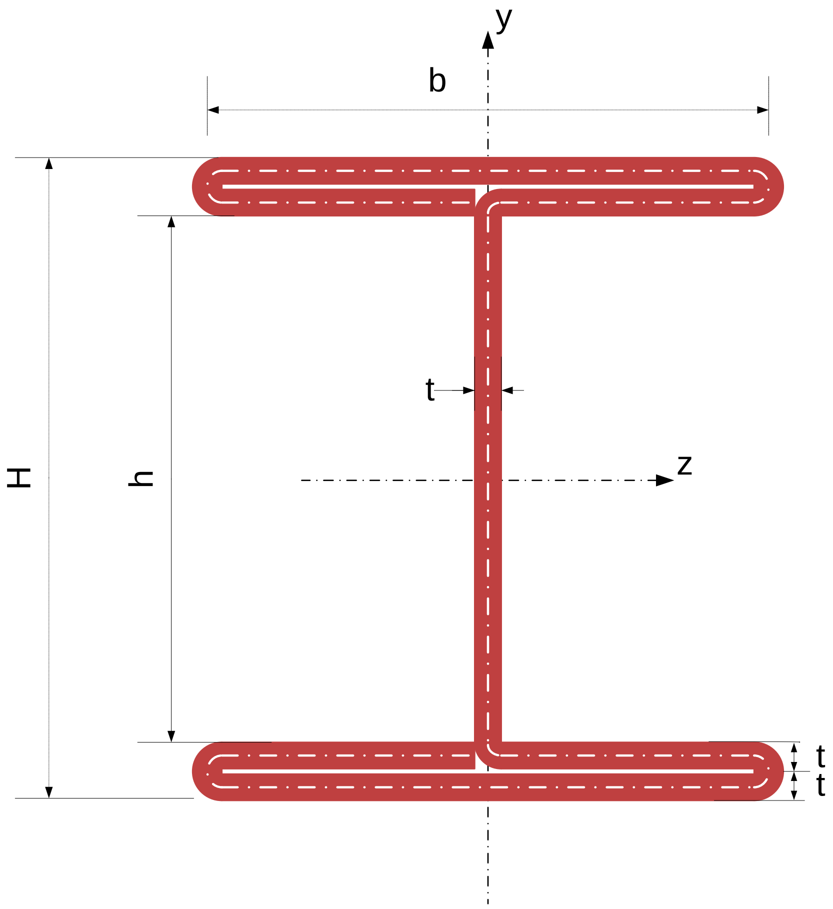

Fig. 1. Mono-symmetrical I section

The Methodology:

- **Statement of the optimization problem:**

The optimization problem is formulated for a simply supported beam loaded with two moments of value *M*, applied to the beam ends (pure bending).

In this first paper the objective function to be optimized is the beam mass. Supposing that the material is homogenous and the beam section is uniform, this function can be replaced by the area of the beam cross section: $F(x_i) = A$

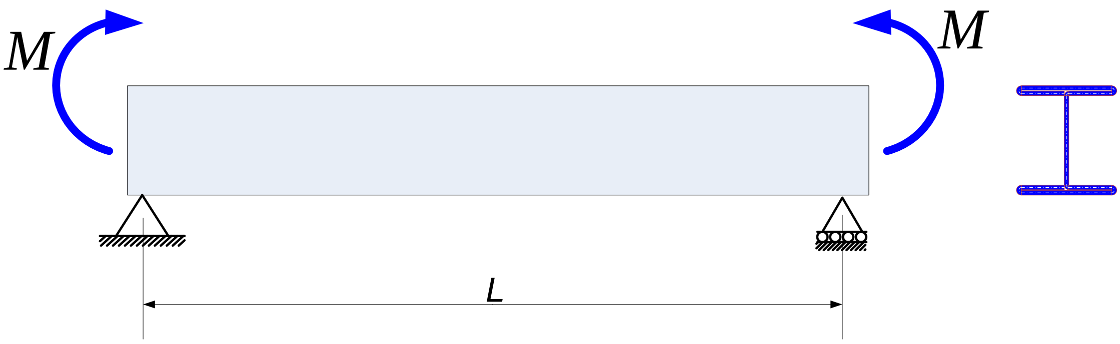

Fig. 2. Simply supported beam loaded with fixed moment at its ends

Generally the sections of cold rolled thin walled beams are formed from a hot rolled steel strips or sheets which are available in the market in different widths and thicknesses. By folding processes of these strips and sheets, the desired cross section can be fabricated.

The design variables of the optimization problem, in this case, will be the cross section dimensions: *h*, *b* in addition to the strip width *B* and its thickness *t*.

Even it's clear that *B & t* are both discrete; we will consider them as continuous variables without loosing the validity of our results.

After choosing the objective function and the design variables, we will discuss the geometrical, technological and structural constraints.

The geometrical constraints must meet the hypothesis of the beam bending theory, so we assume $H/L \leq 0.1$; and they must be adequate with the thin walled beam definition, where we prefer to replace the constraint $H/t \geq 10$ adopted in [2] by the four constraints: $10 \leq h/t \leq 50$, $0.5 \leq b/h \leq 1$ *for* each element of the cross section.

Similarly, the constructional or technological constraints will replace the constraints 2*a*□□□*t*□□□*Hmax* and *b* □□*t* □□*Hmax* adopted in [2] by $4b+h=B$, which reflect better the folding process of doubly symmetric I-beam section shown in figure 3.

In addition to the above mentioned geometrical and constructional constraints, the basic structural constraints [6], of the design of thin-walled structures, i.e. strength and stability constraints are maintained. So the strength constraint is determined by the allowable stress σ_{all} and takes the following simple form.

$$M \leq M_1, \quad \text{with} \quad M_1 = M = \frac{2J_z}{H}.\sigma_{all}$$

where J_z is the moment of inertia of the cross section area, with respect to z axis. And the general stability constraint, as lateral distortional buckling condition may be written in the following form:

$$M \leq M_2, \qquad M_2 = \frac{\pi E}{n_b L}\sqrt{\frac{J_y J_t}{2(1+\nu)}(1+2(1+\nu))\frac{\pi^2}{L^2}\frac{J_\omega}{J_t}}$$

where n_b safety coefficient with respect to general loss of stability.

J_y moment of inertia of the cross section area, with respect to y axes

J_ω warping moment of inertia of the cross-section

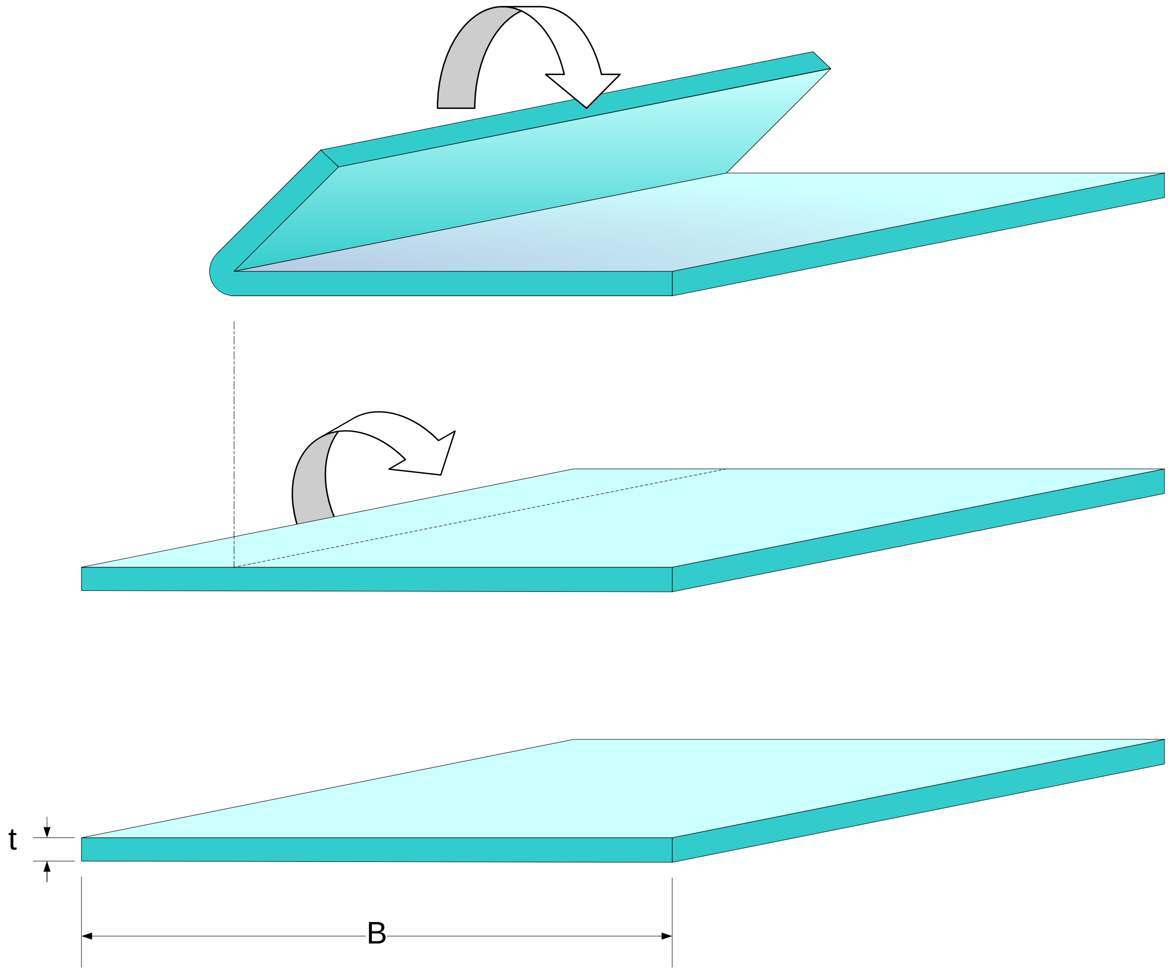

Fig. 3. Folding process of doubly symmetric I-beam section

Finally the local stability constraint may be written in the form:

- first condition of local stability (for the flange)

$$M \le M_3; \qquad M_3 = 2\frac{J_z}{n_{bl}H}.\sigma_{\max.cr}$$

$$\sigma_{\max.cr} = \frac{\pi^2 E}{3(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

- second condition of local stability (for the web)

$$M \le M_4; \qquad M_4 = 2\frac{J_z}{n_{bl}H}.\sigma_{\max.cr}$$

$$\sigma_{\max.cr} = \frac{\pi^2 E}{2(1-\nu^2)}\left(\frac{t}{a}\right)^2 \qquad \text{where } a = (h+3t)/2$$

After discussing the objective function, and all the constraints we can summarize our problem in the following standard mathematical form as follow:

$$\min f(x) = (4b+h)t$$

$$\text{subject to}\begin{cases}(c_1) \ \ 4b+h=B \\ (c_2) \ \ 10 \le \dfrac{h}{t} \le 50 \\ (c_3) \ \ \dfrac{1}{2} \le \dfrac{b}{h} \le 1 \\ (c_4) \ \ \dfrac{L}{H} \le 10 \\ (c_5) \ \ M \le M_1, \ \ M_1 = M = \dfrac{2J_z}{H}.\sigma_{all} \\ (c_6) \ \ M \le M_2, \ \ M_2 = \dfrac{\pi E}{n_b L}\sqrt{\dfrac{J_y J_t}{2(1+\nu)}(1+2(1+\nu))\dfrac{\pi^2}{L^2}\dfrac{J_\omega}{J_t}} \\ (c_7) \ \ M \le M_3, \ \ M_3 = 2\dfrac{J_z}{n_{bl}H}.\sigma_{\max.cr} \\ (c_8) \ \ M \le M_4, \ \ M_4 = 2\dfrac{J_z}{n_{bl}H}.\sigma_{\max.cr}\end{cases}$$

- **Numerical treatment:**

It's clear from the above formulation especially constraints no.(5,6,7,8) that the problem is nonlinear, and it is very hard to get the derivatives of the constraint functions with respect to the design variables as the former are implicit functions of the latter. So, we will not use a gradient based method for solving this problem, but we will adopt an artificial intelligence technique based on the mechanism of natural selection called the Genetic Algorithm (GA).

GAs differ from traditional search techniques in several ways [7]:

- GAs do not require problem specific knowledge to carry out a search.
- GAs use stochastic operators rather than deterministic operators and appear to be robust in noisy environments.
- GAs operate on multiple partial solutions simultaneously (sometimes called implicit parallelism), gathering information from a population of search points to direct subsequent search efforts. Their ability to maintain multiple partial solutions concurrently helps make GAs less susceptible to the problems of local maxima and noise.

We have written a Matlab code which calls a GA built in function. A rapid survey of the influence of the GA parameters, i.e. population size, maximum number of generations, mutation and crossover fraction has shown that this last plays the essential role in improving the convergence of the problem solution. To overcome the difficulty of tuning this parameter, we have rerun the code for varying values of it to choose the one suite our problem. The following figure determines the best crossover fraction occurring when the objective function has its minimum value (span = 3500 mm).

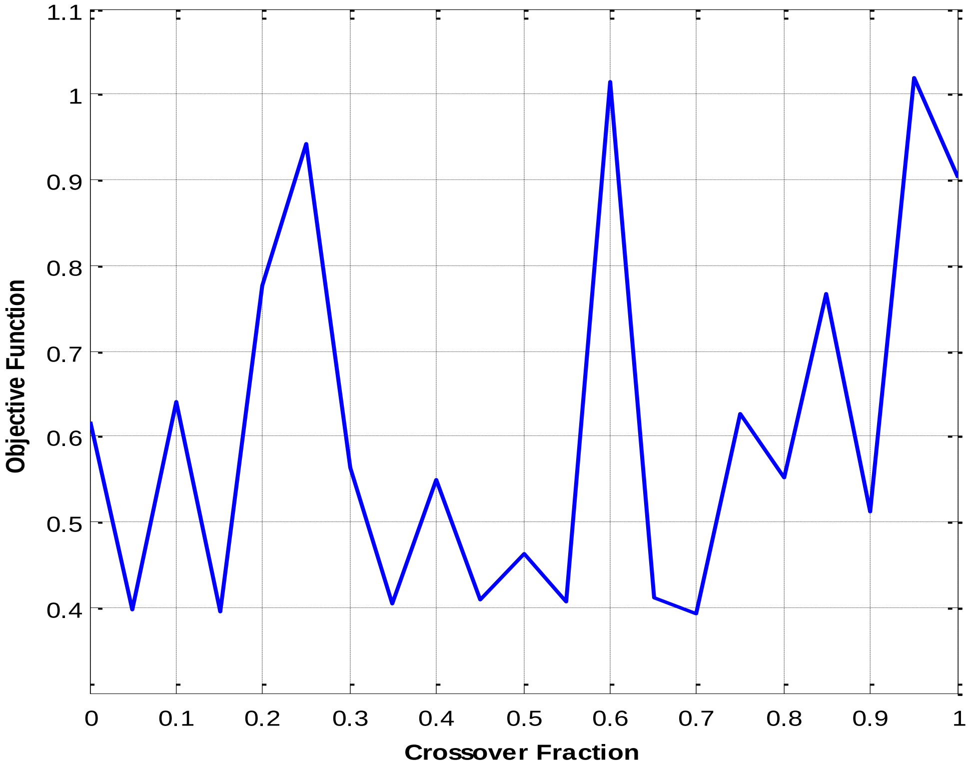

Fig. 4. Variations of the objective function with crossover fraction

For a constant value for the external applied moment and a varying span, we have determined the minimum value of the objective function then we traced the diagram shown in figure.5., giving the variation of the objective function with the span.

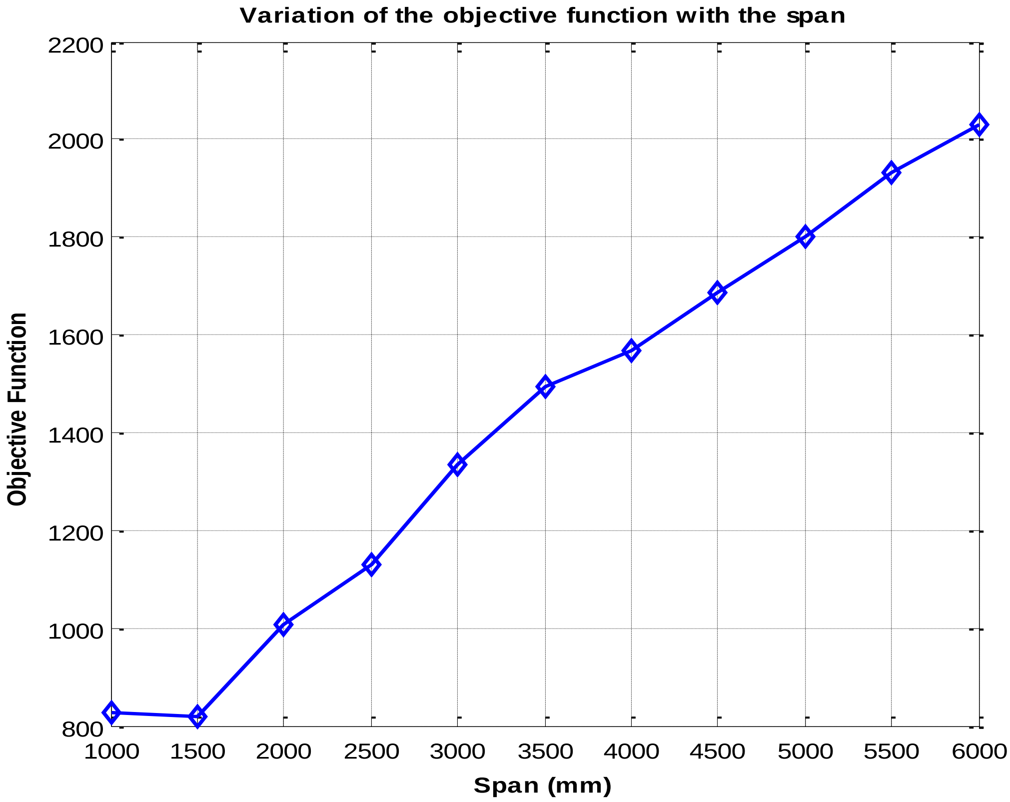

Fig. 5. Variation of the objective function with the span length

In general, we can see that the objective function is increasing with the span reflecting the lower bounded constrained ratio between the total depth of the section and the span.

The above mentioned pattern of variation is monotone everywhere except around the value (1500), where we have observed a changing in the dominance of constraints.

In reality, for the span L=1000, the geometrical, constructional and the beam bending hypothesis constraints in addition to the M_1 strength constraint were largely dominant.

For the other values, the M_2 general stability constraint becomes dominant.

Results and Discussion:

- **Verification of numerical results:**

A global comparison of our numerical results with those obtained in [2] was very satisfactory as we can see in figure. 6.

The differences between our results indicated by (∗) linked by the blue line and their results represented by the (◊) linked by red line can be explained by our modifications of the geometrical and constructional constraints.

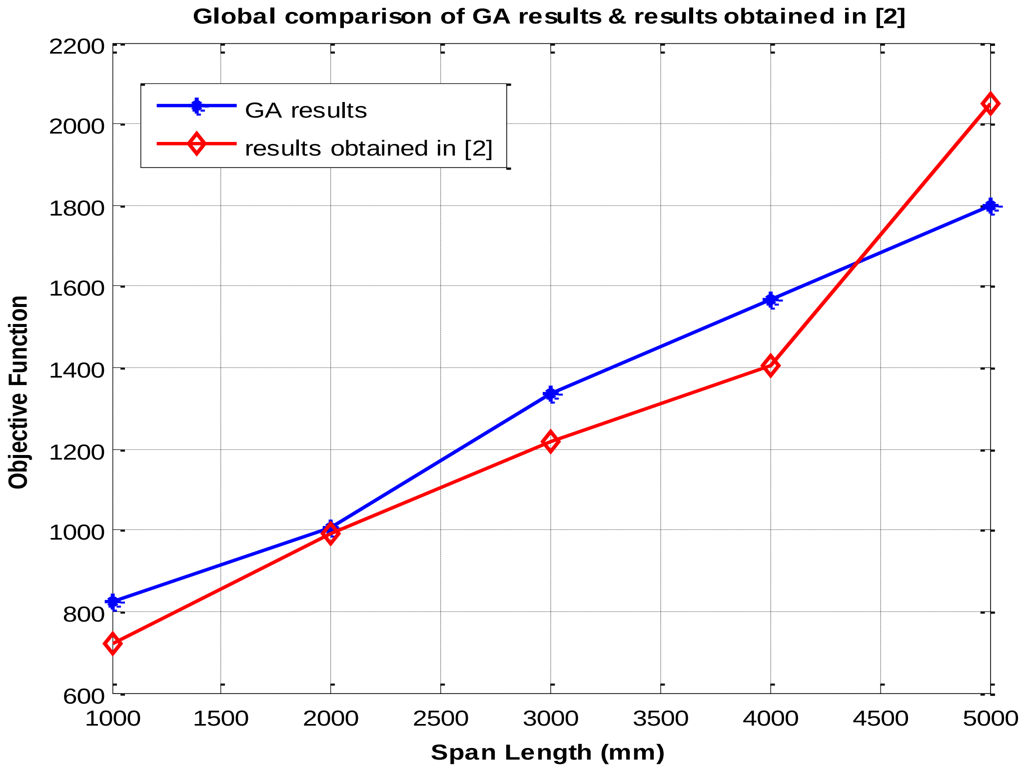

Fig. 6. Global Comparison of GA results & results obtained in [2]

For further verification of our results, we have compared the obtained results with the graphical solution of a reduced optimization problem. Hereafter, we will call the non-reduced problem, as the 4 variables original problem, while the reduced one will be called as the 2 variables reduced problem. In the reduced problem we fixed the values of the strip dimensions for every beam span as indicated in the following table.

These fixed values were inspired by the solution of the 4 variables version of the specified span as one can see on the following tables.

Tab. 1. Strip widths and thicknesses for the reduced problem

Span Length (mm)	Strip thickness (mm)	Strip width (mm)
1000	2	450
1500	2	430
2000	3	470
2500	3	530
3000	3	550
3500	3	570
4000	3	620
4500	3	640
5000	3	660
5500	3	680
6000	3	700

Choosing the case of L=4000 mm, substituting the corresponding fixed values in the problem formulation and resolving for the optimum solution using the same GA program we obtained a multitude of optimum solutions all having the same objective function value.

Tab. 2. Numerical results of the reduced problem for L=4000mm

h (mm)	b (mm)	t (mm)	B (mm)	**A (mm^2)**
125.4477	123.6381	3	620	**1860**
125.595	123.6011	3	620	**1860**
126.552	123.3621	3	620	**1860**
126.8607	123.2847	3	620	**1860**
127.17	123.2075	3	620	**1860**
. . . .	. . . .	. . . .	. . . .	. . . .
149.2171	117.6957	3	620	**1860**
149.2237	117.6942	3	620	**1860**
149.363	117.6594	3	620	**1860**
149.4138	117.6464	3	620	**1860**
149.5797	117.6052	3	620	**1860**

This fact was confirmed by the graphical solution (possible only in the case of 2 variables problems). Where the feasible region is restricted to a linear segment containing all the points represented by the optimum solutions (*b*,*h*). The general constraints of the optimization problem are transformed into plane curves or straight lines that appear on the figure no. 7, in the following order:

- g_1 represents the line expressing constraint c_1.
- g_2 & g_3 represent constraint c_2.
- g_4 & g_5 represent constraint c_3.
- And finally g_6 represents constraint c_4.

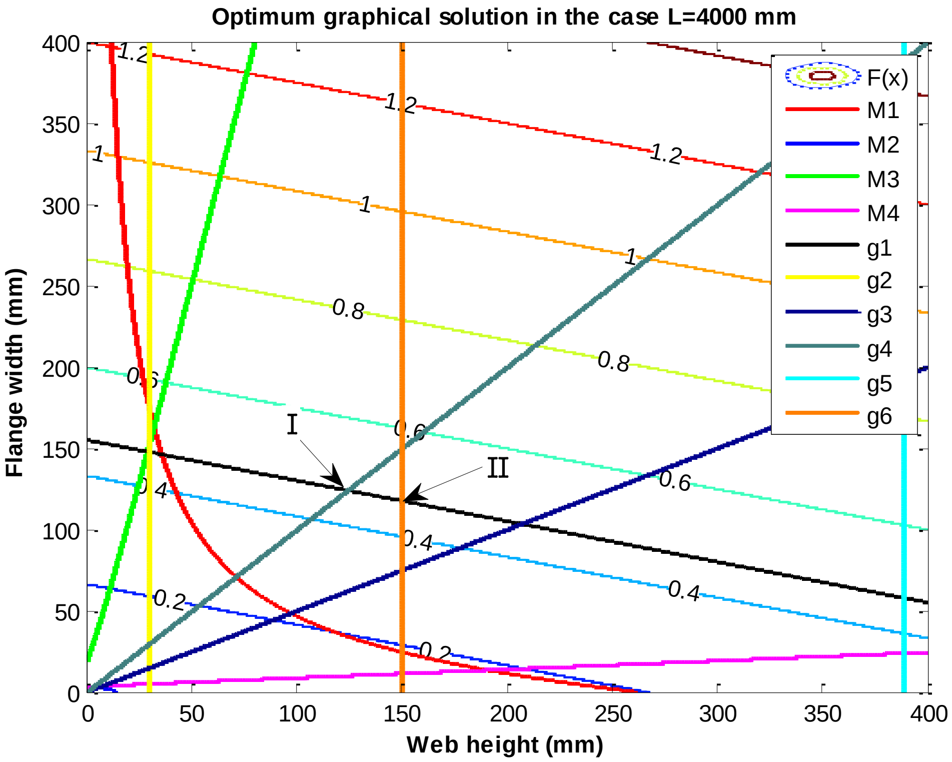

Fig. 7. Graphical solution in the case L = 4000 mm

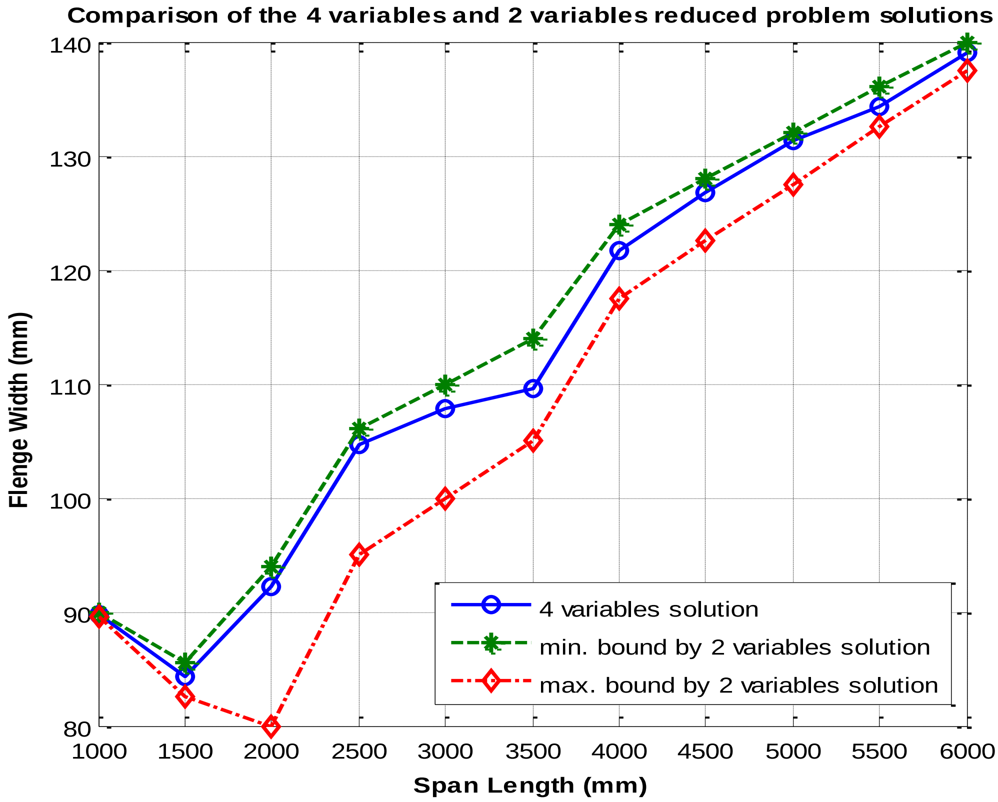

Fig. 8. Comparison of the 4 variables & 2 variables reduced problem solutions (Flange width)

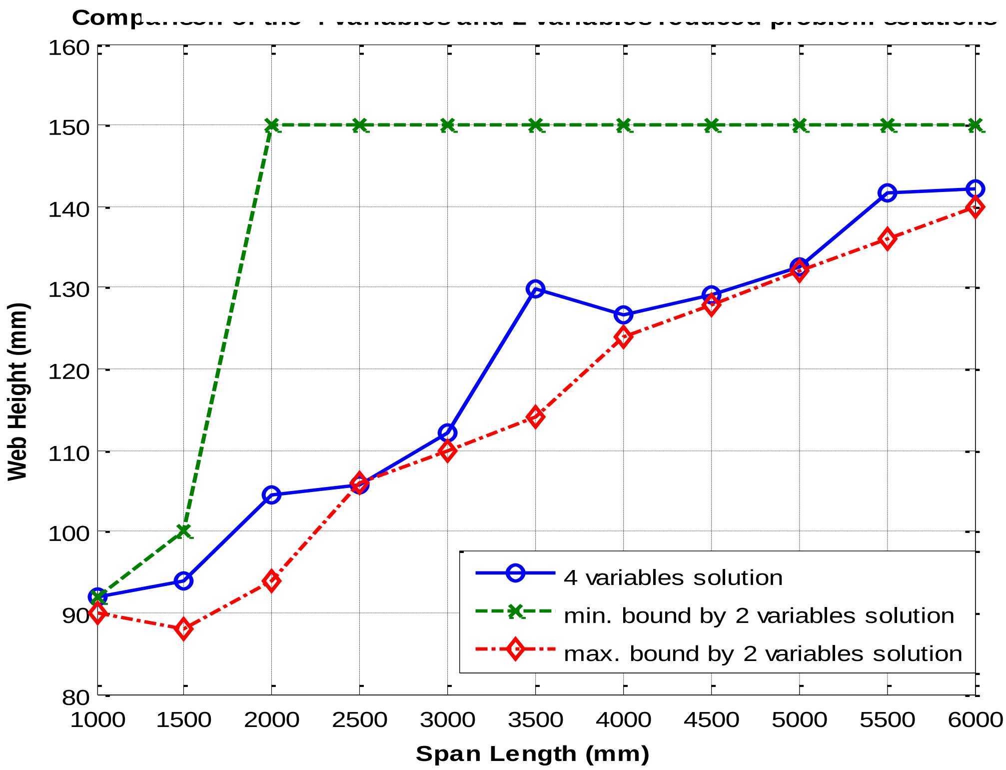

Fig. 9. Comparison of the 4 variables & 2 variables reduced problem solutions (Web height)

At figure no. 8 we can observe that the optimum values of the flange width, given by the GA Matlab solution of the 4 variables problems, are located between the minimum and maximum values corresponding to points II & I respectively.

At figure no. 9 we can observe that the optimum values of the web height, given by the GA Matlab solution of the 4 variables problems, are located between the minimum and maximum values corresponding to points I & II respectively.

The optimality of these solutions is confirmed by the fact that the solution is closer to points I.

Conclusions and Recommendations:

In this paper we have readjusted the formulation of an optimum structural design problem in order to make it closer to the real conditions of the engineering practice. In reality, we have replaced the irrational geometrical constraints by more adequate ones inspired by the availability of the steel sheets and strips in the market. Readjusting the problem formulation makes it more ready to be directly implemented in structural engineering software.

The new constraints in our modified formulation have imposed the use of non-gradient based optimization techniques; among these techniques we have chosen the genetic algorithm method for its simplicity and easiness of its implementation in the aimed software. In addition we have examined the suggested algorithm, i.e., GA Matlab tool, by an appropriate way which has shown that it is a good tool to solve such a problem.

Further investigation can be done to evaluate the effect of different loading cases and different support types.

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