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Influence of the Main Support Types on the Optimum Design of a Cold Formed Mono-symmetrical I Beam under the Action of Uniformly Distributed Load

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□ ABSTRACT □

In this paper we present a modern treatment of the optimum design procedure for a cold formed mono-symmetrical I beam under the action of uniformly distributed load. This problem is highly governed by different kinds of buckling constraints. As the walls of this beam are very thin, we will consider the design constraint of the global lateral buckling in addition to the design constraints related to the local buckling of the flanges and the web. This constraint will be established for four cases of vertical support conditions and only one case of lateral support condition, and as these two groups of support conditions could be practically independent, we suppose that the shape of the lateral buckling is not altered by the vertical displacement shape. Then, we investigate the influence of the vertical displacement boundary conditions dictated by the type of the vertical supports on the values of the critical lateral buckling load. The other constraints of the problem are related to the fabrication processes. This optimum design problem will be solved using a Genetic Algorithm program implemented using MATLAB. The test of this program constitutes an essential part of this paper.

Key words: lateral buckling, main support types, structural constraints, optimum structural design, thin walled beams, genetic algorithm

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تأثير نمط الاستناد الرئيسي على التصميم الأمثل لجائز I نظامي وملفوف على البارد تناظرياً تحت تأثير حمولة موزعة بانتظام

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□ ملخّص □

نقدم في هذه المقالة مسألة التصميم الإنشائي الأمثل لجائز I نظامي وملفوف على البارد تناظرياً تحت تأثير حمولة موزعة بانتظام. تتأثر هذه المسألة بأشكال متنوعة من التحنيب (عدم الاستقرار) وذلك بسبب قلة سماكة الصفيحة المشكلة للمقطع. بالإضافة إلى قيود التصميم المتعلقة بالتحنيب الموضعي للجناحين وللجسد، سنأخذ بعين الاعتبار قيد تصميم متعلق بالتحنيب الكلي الجانبي. سنستنتج عبارة هذا القيد لأربع حالات من أنماط الاستناد الشاقولي ولنمط واحد من الاستناد الجانبي. وبما أن هذين النمطين مستقلان عملياً، سنفترض أن شكل نمط التحنيب الجانبي ثابت ولا يتأثر بشكل الانتقال الشاقولي، ثم سندرس تأثير شروط الانتقال الشاقولي الطرفية على قيم حمولة التحنيب الكلي الجانبي الحرجة. أما بقية قيود التصميم فهي متعلقة بآلية تشكيل المقطع. وستحل مسألة التصميم الأمثل الناتجة باستخدام خوارزمية جينية سيتم في هذه المقالة استعراض نتائجها واختبارها.

الكلمات المفتاحية: التحنيب الجانبي، نمط الاستناد الرئيسي، قيود التصميم الإنشائية،التصميم الإنشائي الأمثل، الجيزان رقيقة الجدران، الخوارزميات الجينية.

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Introduction:

One of the problems that the structural designer of thin-walled beams has to tackle is its sensitivity to instability phenomena. This problem becomes more difficult when the sections are cold formed by folding sheets or strips. In this case the designer has to define the section using geometric variables able to describe all the formed folds. Then he has to choose these variables in order to minimize the cost and to verify the technological and structural constraints[1]. Among the latter, the local and global buckling constraints become very complicated and difficult to be determined without using energy approximation methods[2]. Regarding the complexity of these constraints, the classical trial and error procedure of section design becomes quasi impossible and the optimum design procedure imposes itself as the only possible way[3]. In addition the use of gradient based methods of optimization is limited by the nonlinearity of the above mentioned constraints; therefore, in this paper we suggest the use of more suitable Genetic Algorithm method and show its implementation using MATLAB[4].Then the results giving by this algorithm will be validated by studying the variation of the cost with the span of a simple beam under the action of uniformly transverse load having different types of supports.

Importance and aims of the paper

The main aim of this paper is to generalize previously presented procedure of the section shape optimization [5]. This generalization concerns the possibility of dealing with more realistic loading and with different types of vertical supports.

The secondary aim is to test the validity of our Genetic Algorithm in more practical situations less simple than the pure bending loading used in [5].

The Methodology:

In this paper our methodology will be the same as in [5] except that here we feel it necessary to expose our methods and assumptions used to establish the suitable critical load expressions. In reality, as it is known, the loss of stability of thin-walled beams often occurs through a combination of bending and torsion, even if the loading consists only of transverse and axial loads in one plane. The basic types of such instability are the lateral buckling of beams and the axial-torsional buckling of columns.

We start our presentation of the methodology by giving a brief introduction to the study of the lateral buckling of a beam loaded by uniformly distributed load. We will examine several cases of vertical support types (a,b,c,d) considered as shown in fig. 1 while the lateral support types will be the same: pin-pin.

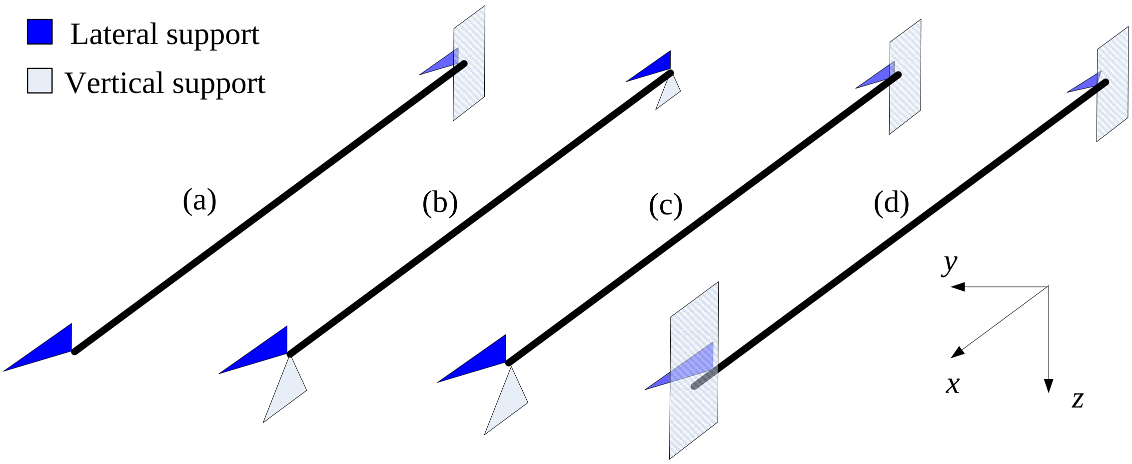

Fig. 1.Vertical & lateral support types

Two of these cases(a & d) are explicitly illustrated by a 3D sketch representing at the left the beam before buckling and at the right the beam after it has buckled as shown in fig. 2 & 3.

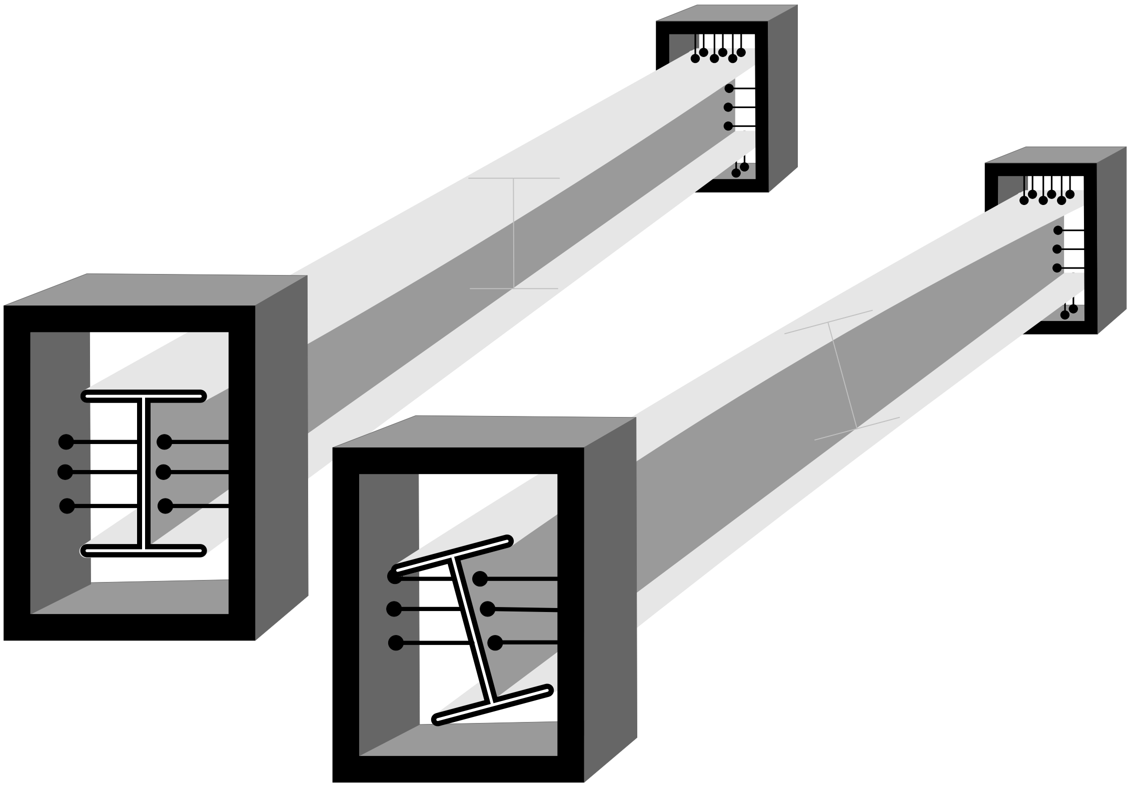

Fig. 2.3D illustration of case (a), the cantilever

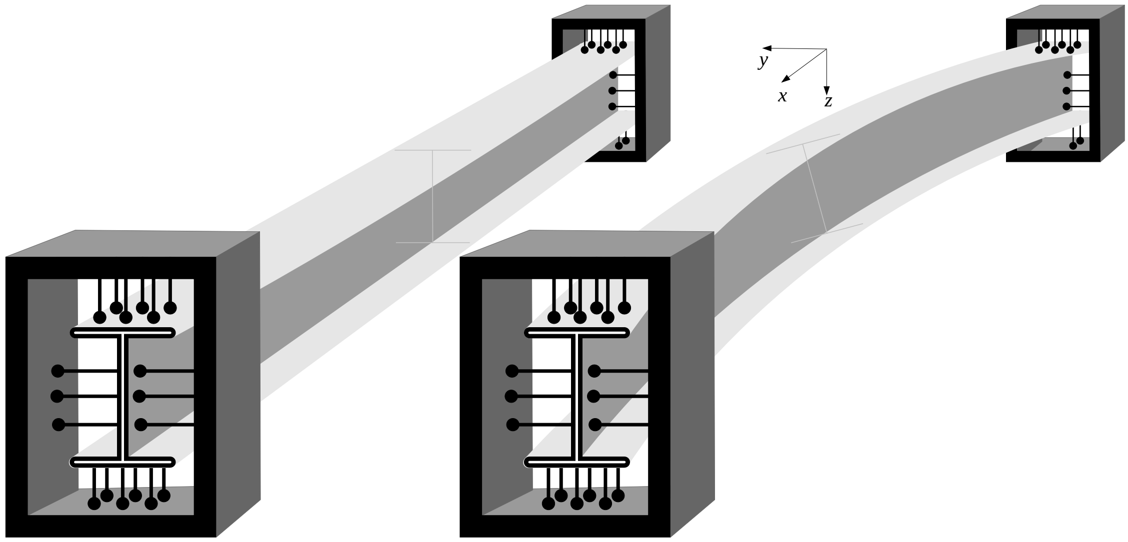

Fig. 3.3D illustration of case (d), fixed-fixed beam

- **Lateral buckling – critical load of the beam:**

Kinematic relationships:

We introduce the increment displacement field in an arbitrary point (*x*,*y*,*z*), where *x* indicates the considered section, as follows[2]:

$$u_1 = u - v'y - w'z - \theta'\omega$$
$$u_2 = v - \theta z$$
$$u_3 = w + \theta y$$

where:

u,*v* and *w* are the displacements of the center of the cross section. They are functions of *x*.

ωis the warping function which is given for the upper flange, bottom flange and the web respectively as follows: $-\frac{hy}{2}, \frac{hy}{2}$ $and\ 0$

θis the angle of twist. It is also a function of *x*.

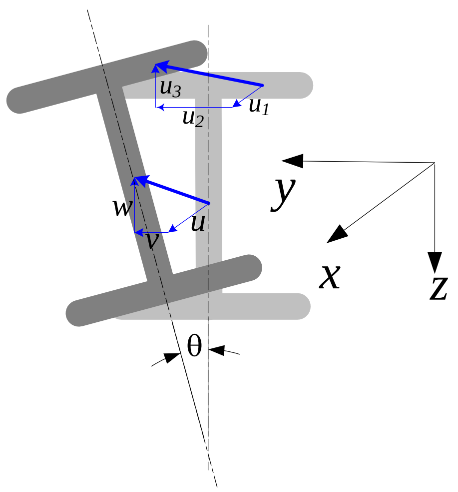

Fig. 4.3D illustration of the displacement field

From this displacement field we deduce that the 1st order approximation of the axial normal strain at any point of the cross section is:

$$e_x = u' - v''y - w''z - \theta''\omega$$

and the only non-zero second order strain components are the following:

$$\varepsilon_x = u' - v''y - w''z - \theta''\omega + \frac{1}{2}(v' - \theta' z)^2 + \frac{1}{2}(w' + \theta' y)^2$$

$$\gamma_{xz} = \left(\frac{\partial u_2}{\partial x}\right)\left(\frac{\partial u_2}{\partial z}\right) = -v'\theta + \theta\theta' z$$

$$\gamma_{xy} = \left(\frac{\partial u_3}{\partial x}\right)\left(\frac{\partial u_3}{\partial y}\right) = w'\theta + \theta\theta' y$$

Potential energy:

Restricting ourselves to the case of a beam under initial axial force *P*, initial lateral load p_z, initial bending moment M_z^0 and initial shear force V_z^0, we can write the strain energy of the initially stressed I beam as [2]

$$U = \int_0^l \int_A \left(\sigma^0 \varepsilon_x + \frac{1}{2} E e_x^2 + \tau_{xy}^0 \gamma_{xy} + \tau_{xz}^0 \gamma_{xz}\right) dA\, dx + \int_0^l \frac{1}{2} GJ\theta'^2 dx$$

in which *l* is the length of the beam, *A* is the cross-section area, σ^0 is the initial normal stress in the cross section, τ_{xy}^0 and τ_{xz}^0 are the initial shear stresses, γ_{xy}, γ_{xz} are the associated shear angles, *G* is the elastic shear modulus, and *GJ* is the torsional stiffness for simple torsion. In the precedent expression, the strain energy associated with the stresses $\sigma_{yy}, \sigma_{zz}, \tau_{yz}$inthe plane of the cross section is negligible.[2]

Computing the initial and current stresses, and then performing the integral over the section, this expression becomes:

$$U = \int_0^l \left[-\frac{P}{A}\left(u'A + \frac{v'^2}{2}A + \frac{\theta'^2}{2}I_z + \frac{w'^2}{2}A + \frac{\theta'^2}{2}I_y\right) - \frac{M_z^0}{I_z}\left(-w''I_z - v'\theta' I_z\right) + M_z^{0\prime} v'\theta + \frac{E}{2}\left(u'^2 A + v''^2 I_y + w''^2 I_z + \theta''^2 I_\omega\right) + \frac{GJ}{2}\theta'^2 \right] dx$$

Where the following geometric properties of the cross section $A, I_y, I_z \; and \; I_\omega$ are respectively the area, moment of inertia about *z*, *y* and the warping moment of inertia.

The load work expression is given by:

$$W = \int_0^l \left[(p_z + \Delta p_z)w + \Delta p_y v + \Delta m_t \theta\right] dx + [\Delta M_z w']_0^l + [\Delta M_t \theta]_0^l - [(P + \Delta P)u]_0^l$$

in which $\Delta p_y$and $\Delta p_z$are small transversal distributed disturbing loads (added tothe initial loadp_z), $\Delta m_t$is a small perturbation representing an applied distributed momentabout the beam axis, $\Delta M_y$and$\Delta M_t$are small moments about the *z* and *x* axes applied at the beam ends, and $\Delta P$is a small disturbing incrementin the axial compressive load.

The point of application of $p_z$in the previous equation is at the beam centroid. If p_z is applied on the top flange, then the integrand in this equation must be augmented by the term $-p_z\left(\frac{1}{4}h\theta^2\right)$ because rotation θ causes additional negative displacement $\frac{1}{4}h\theta^2$.

Regrouping the terms in the above two equations, we get the full expression for the potential energy $\Pi = U - W$.From this expression of the potential energy we could deduce the differential equations and the boundary conditions as one can see in [2].The closed solution of these equations is possible only for simple sections [6], but rather impossible for more complicated profiles like the I-section, so we prefer to use approximate variational method of solution.

In the case of our problem, the lateral buckling of beams due to transversal load function $p_z(x)$, we have $(P = 0,\ u' = 0,\ w = 0)$, if we let the bending moment distribution be described as $\overline{M}_z^0(x) = p_z m(x)$where *m(x)* is the bending moment function for the case of the unit uniformly distributed load. The upper bound approximation of the critical value of the load $\overline{p}_z$that causes the lateral buckling can be found using the Raylieigh quotient expression deduced from the stability condition given in [2] , as

$$\overline{p}_{zcr} = \frac{\int_0^l \frac{1}{2}(EI_y v''^2 + EI_\omega \theta''^2 + GJ\theta'^2)dx}{\int_0^l v''\theta\, m(x)dx}$$

In this approximation method it's usually assumed that the shape of lateral buckling and the angle of twist are known, but their values are dependent of two multiplicative constants q_1, q_2 . For all cases of support types, we will use the two expressions:

$$v = q_1 \sin\frac{\pi x}{l} \text{ and } \theta = q_2 \sin\frac{\pi x}{l}$$

Then, for the considered cases of support types, the bending moment function *m(x)* and the resulted upper bound value of the critical load $\overline{p}_{zcr}$ are listed here down:

- Cantilever Beam under uniformly distributed transversal load p_z: in this case from the static equations: $m(x) = \frac{1}{2}(l - x)^2$substituting in the above integrals we get

$$\overline{p}_{zcr} = \frac{12\pi^3}{l^4(2\pi^2 - 3)}\sqrt{EI_y(EI_\omega\pi^2 + GJl^2)}$$

- Simply Supported Beam under uniformly distributed transversal load p_z: in this case from the static equations: $m(x) = \frac{1}{2}x(l - x)$ substituting in the above integrals we get

$$\overline{p}_{zcr} = \frac{12\pi^3}{l^4(\pi^2 + 3)}\sqrt{EI_y(EI_\omega\pi^2 + GJl^2)}$$

- Fixed-Pinned Beam under uniformly distributed transversal load p_z: in this case from the static equations: $m(x) = -\frac{l^2}{8}\left(1 - 5\frac{x}{l} + 4\frac{x^2}{l^2}\right)$ substituting in the above integrals we get

$$\overline{p}_{zcr} = \frac{48\pi^3}{l^4(\pi^2 + 12)}\sqrt{EI_y(EI_\omega\pi^2 + GJl^2)}$$

- Fixed-Fixed Beam under uniformly distributed transversal load p_z: in this case from the static equations: $m(x) = -\frac{l^2}{12}\left(1 - 6\frac{x}{l} + 6\frac{x^2}{l^2}\right)$ substituting in the above integrals we get

$$\overline{p}_{zcr} = \frac{4\pi^3}{l^4}\sqrt{EI_y(EI_\omega\pi^2 + GJl^2)}$$

The comparison between the four obtained expressions shows that the beam resistance to the buckling load matches very well with support types.

- **Statement of the optimization problem:**

The optimization problem is formulated for the four previous cases as follows:

$$\min f(x) = (4b+h)t$$

$$\text{subject to} \begin{cases} (c_1) \quad 4b+h=B \\ (c_2) \quad 10 \le \dfrac{h}{t} \le 50 \\ (c_3) \quad \dfrac{1}{2} \le \dfrac{b}{h} \le 1 \\ (c_4) \quad \dfrac{L}{H} \le 10 \\ (c_5) \quad p \le p_1, \quad p_1 = \dfrac{2I_z}{Hm_{\max}}.\sigma_{all} \\ (c_6) \quad p \le p_2, \quad p_2 = p_{crz} \\ (c_7) \quad p \le p_3, \quad p_3 = 2\dfrac{I_z}{n_{bl}H}.\sigma_{\max.cr} \\ (c_8) \quad p \le p_4, \quad p_4 = 2\dfrac{I_z}{n_{bl}H}.\sigma_{\max.cr} \end{cases}$$

Where the geometric parameters defining the optimization problem are illustrated in the following figure.

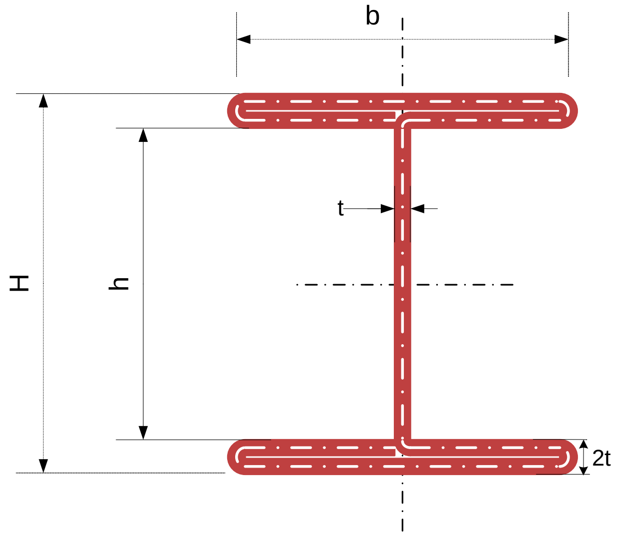

Fig. 5.Cross section geometric parameters

The geometrical and constructional constraints $(c_1 \to c_4)$are taken as in [5], while the strength and stability constraints$(c_5 \to c_8)$ have been modified to fit with the load and support types as can be shown in the following table.

Tab. 1.Maximum bending moment & approximated critical lateral buckling load

Beam	m_{max}	P_{crz}
Cantilever	$ql^2/2$	$\dfrac{12\pi^3}{l^4(2\pi^2-3)}\sqrt{EI_y(EI_\omega\pi^2+GJl^2)}$
Simply supported	$ql^2/8$	$\dfrac{12\pi^3}{l^4(\pi^2+3)}\sqrt{EI_y(EI_\omega\pi^2+GJl^2)}$

Fixed-Pinned	$ql^2/8$	$\frac{48\pi^3}{l^4(\pi^2+12)}\sqrt{EI_y(EI_\omega\pi^2+GJl^2)}$
Fixed-Fixed	$ql^2/12$	$\frac{4\pi^3}{l^4}\sqrt{EI_y(EI_\omega\pi^2+GJl^2)}$

▪ **Numerical treatment:**

As the nonlinearity of the problem is clear, it's well known that the Genetic Algorithm is a suitable numerical method to find the optimum solution of the problem. The load cases and the support types considered in this problem have implied more complicated constraints. This fact will be a good occasion to truly examine the efficiency of the MATLAB program written and implanted by us. [5]

As it was done in the above mentioned paper, the examination of the algorithm with the program will be done by looking at the variation of the section area (objective function) and some of its dimensions and geometrical properties with the span for the four cases of support types.

Results and Discussion:

The following figures show the results obtained from the mentioned program. Figure no.6 shows the variation of the section area, considered as the objective function of the optimization problem, with the span for the four types of support. The nonlinear variation of this area is very well approximated by a quadratic function as one can see by the dashed line. While the variation of *t* and *h* figs.7&8is more closer to the linear approximation. In addition, figs. 9&10 show the variation of the section properties I_z, I_ω with the span, the examination of their variation shows that they are better approximated by polynomials of even degree higher than the second degree.

All these variations and their approximations are consistent with our elementary knowledge of strength of material .This consistency of the results gives us entire satisfaction concerning the optimization algorithm and the written program.

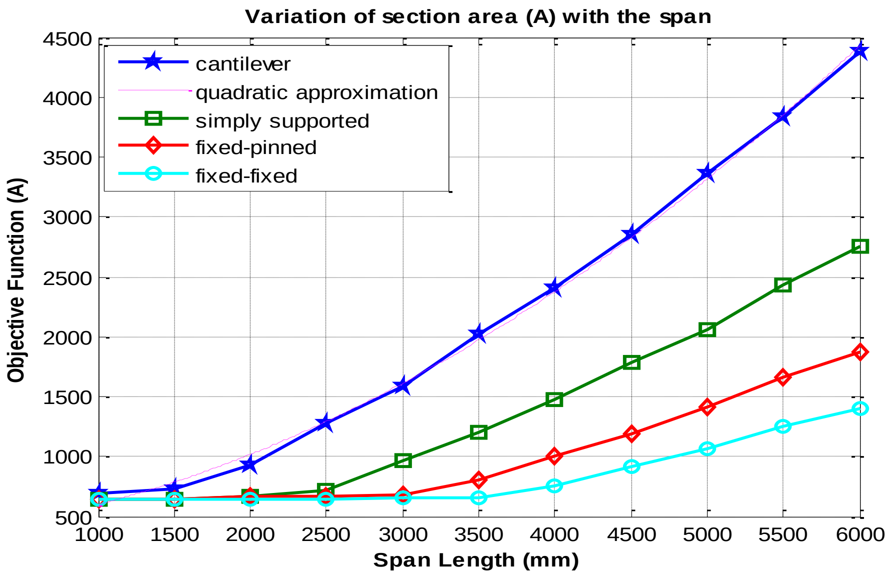

Fig. 6.Variation of section area with the span

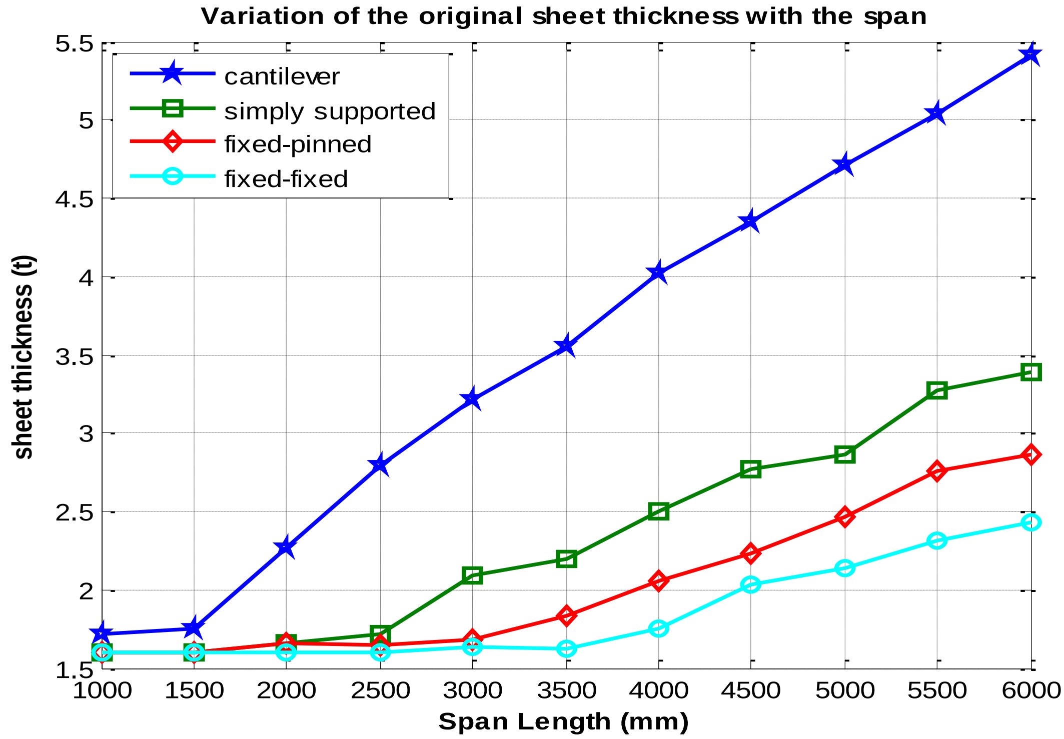

Fig. 7.Variation of the original sheet thickness with the span

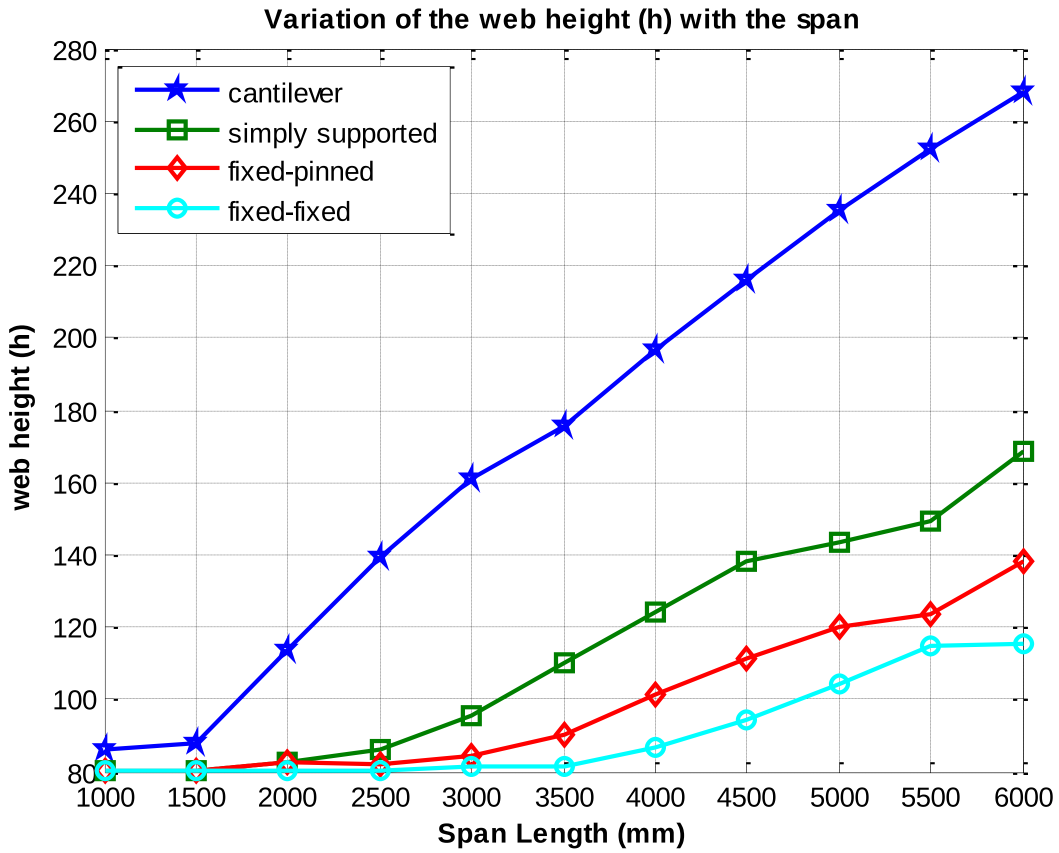

Fig. 8.Variation of the web height with the span

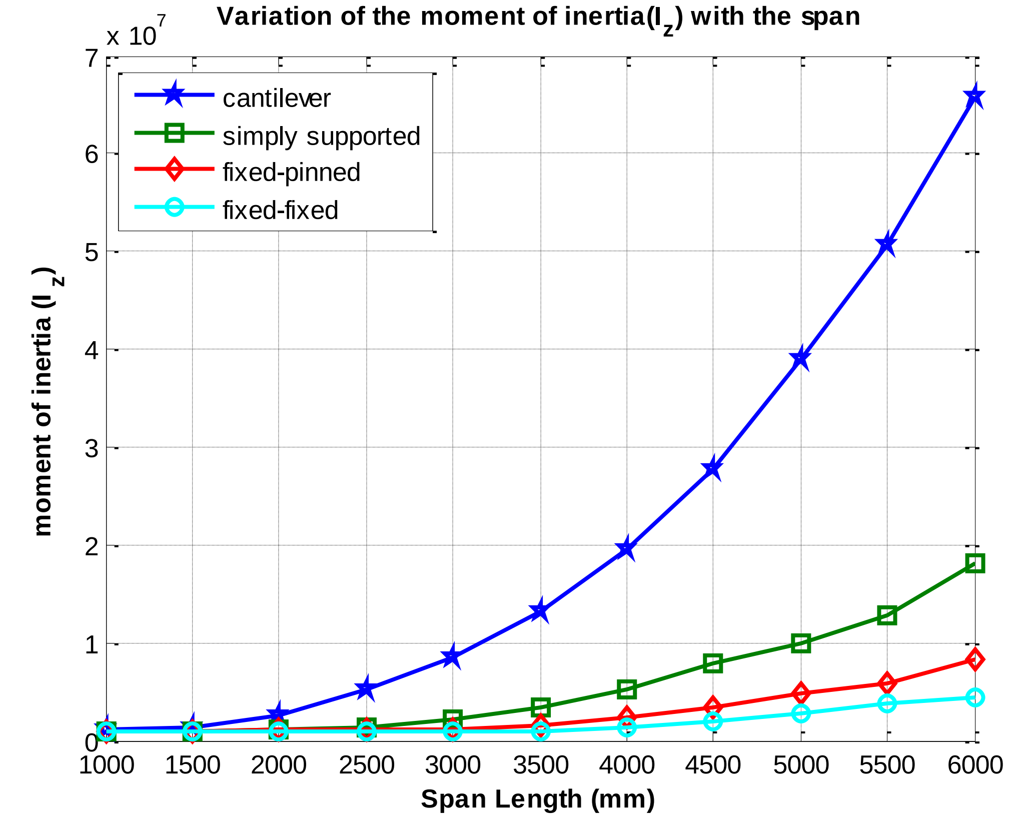

Fig.9. Variation of the moment of inertia with the span

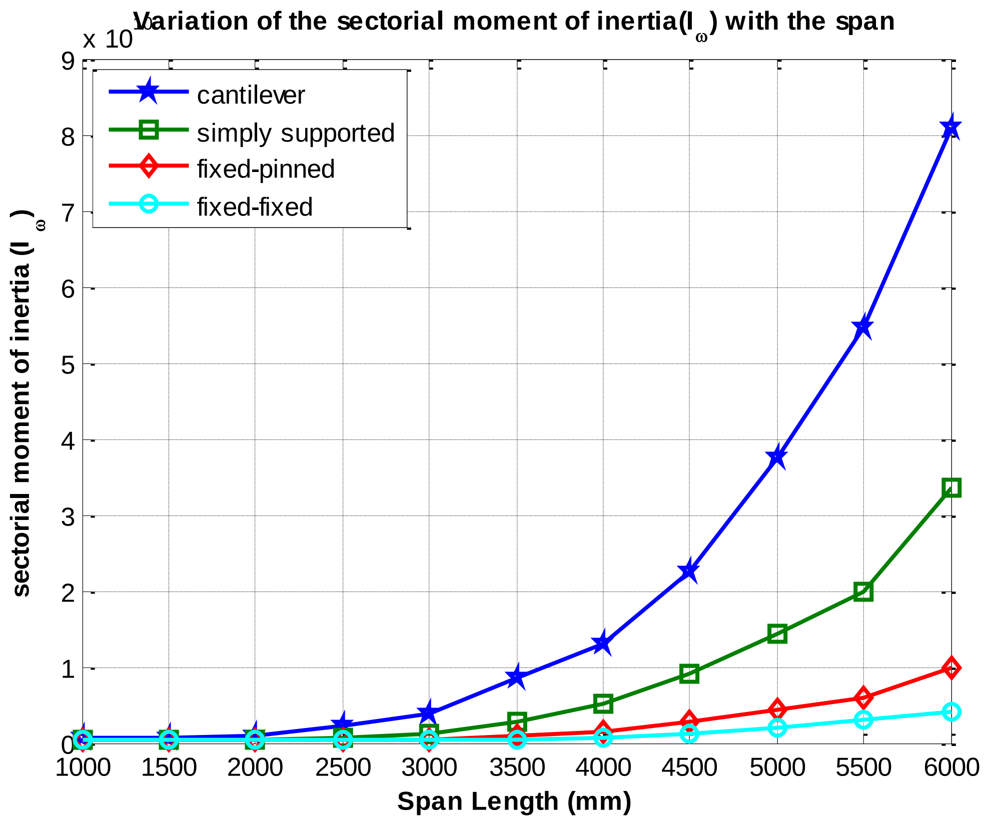

Fig. 10.Variation of the sectorial moment of inertia with the span

Conclusions and Recommendations:

In this paper we have fixed as major aims the generalization of the previously presented procedure of the section shape optimization [5], and the validation of our Genetic Algorithm in more practical situations less simple than the pure bending loading used in [5]. To make the achievement of these two aims more realistic, we chose to obtain the optimum design under the most common load case, uniformly distributed load, and different support types for which we have been able to establish the stability criterion. The obtained results have shown that the above mentioned major and secondary aims have been successfully reached.

To make these results more useful to the practice engineer, it's recommended to examine the effect of different lateral buckling shapes. In reality, these shapes reflect the nature of real structural connections which are more and more modelized as semi-rigid connections by researchers' community.

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