

Feature Extraction in Digital Images (Faces) using 2-D Wavelet Transform.

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□ ABSTRACT □

Recognizing and detecting human faces automatically in off-line photo archive or video frame data provides us with powerful tools for performing queries. In this article, a new scheme for face feature extraction in colored images of frontal poses, using the 2-D wavelet multiresolution decomposition is presented. Each bounded face is described by a subset of band filtered images representing the wavelet coefficients for the three channels red, green, and blue. These coefficients characterize the face texture and outlay, and a set of simple statistical measures allows us to form compact and meaningful feature vectors.

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استخلاص الصفات المميزة للوجه في الصور الرقمية باستخدام تحويل 2-D wavelet ثنائي البعد.

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□ الملخص □

إن أتمتة التعرف والكشف على وجه الإنسان من المعطيات الصورية اللحظية أو المؤرشفة، تتيح لنا إمكانية إجراء تقصيات و تحريات لحظية فعالة. في هذه المقالة، سوف نقدم طريقة جديدة لاستخلاص التفاصيل المميزة في صور الوجوه الملونة عن طريق تحليلها بوساطة تحويل 2-D wavelet ثنائي البعد. حيث يتم توصيف كل صورة وجه بوساطة مجموعة من الصور الجزئية المرشحة والتي تحتوي على معاملات تحويل 2-D wavelet للقنوات الثلاث الأحمر والأخضر والأزرق في تلك الصورة الملونة. إن معاملات التحويل هذه تصف تخطيط الوجه والبنية المميزة له، ونستطيع باستخدام طرق إحصائية بسيطة الحصول على أشعة تفاصيل (vectors) محدودة العدد ومعبرة، من مجموعة المعاملات السابقة. إن الطريقة المقترحة مكونة من جزئين: جزء منهجي يمنح النظام مناعة ضد التغيرات الناتجة عن الإزاحة والدوران بزواوية (10،-10) بحيث يتم تحديد الوجه فقط (استثناء شعر الوجه الأذنان العنق) والجزء الثاني يقوم باستخلاص السمات المميزة للوجه، وذلك تحت شروط وتقييم ثابتة عن طريق استخدام 2Dwavelet متعدد الدقة والذي يتصف بكفاءة حسابية ونتيجة لذلك يكون النظام المقترح صلباً وفعالاً.

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Introduction

Pattern recognition in general and face recognition in specific has become a very important tool in many security based systems like banking systems, police archives, airports ...etc. The video raw data is first automatically segmented into shots and from the content-related image segments, salient features such as region shape, intensity, color, and texture descriptors are extracted and used for indexing and retrieving information. In order to allow queries at a higher semantic level, some particular pictorial objects have to be detected and exploited for indexing. In recent years, considerable progress has been made on the problem of face feature extraction for later detection and recognition, especially under the stable conditions such as small variations in lighting, facial expression and poses. A good survey may be found in [1]. These methods can be roughly divided into two different groups: geometrical features matching and template matching. In the first case, some geometrical measures about distinctive facial features such as eyes, mouth, nose and chin are extracted ([2]). In the second case, the face image, represented as a two-dimensional array of intensity RGB values, is compared to a single or several templates representing a whole face. The earliest methods for template matching are correlation-based, thus computationally very expensive and require great amount of storage and since a few years, the Principal Components Analysis (PCA) method also known as Karhunen-Loeve method, is successfully used in order to perform dimensionality reduction ([3, 4, 5, 6, 7]). In this paper, we propose a new method for face feature extraction based on a 2-D wavelet decomposition of the face images. Each face image is described by a subset of band filtered images containing wavelet coefficients for the 3 channels RGB. From these wavelet coefficients which characterize the face texture and outlay, we form compact and meaningful feature vectors, using simple statistical measures (coefficient's mean values and the deviation values). Then, an Artificial Neural Network based system can be used in order to classify the face feature vectors into person classes.

The Wavelet Transform

In the last decade, wavelets have become very popular, and new interest is rising on this topic. The main reason is that a complete framework has been recently built ([8, 9]) in particular for what concerns the construction of wavelet bases and efficient algorithms for its computation.

The main characteristic of wavelets (if compared to other transformations) is the possibility to provide a multiresolution analysis of the image in the form of coefficient matrices. Strong arguments for the use of multiresolution decomposition can be found in psycho visual research, which offers evidence that the human visual system processes the images in a multi scale way. Moreover, wavelets provide a spatial and a frequential decomposition of the image at the same time. Wavelets are also very flexible: several bases exist, and one can choose the basis which is more suitable for a given application.

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information see fig(1).



Fig (1) shows the time and scale (frequency) relationship in wavelet transform.

Fig (2) what this looks like in contrast with the time-based, frequency-based, and STFT views of a signal:

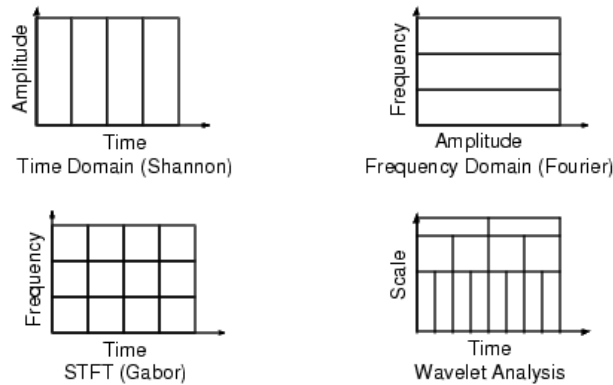


Fig (2) Time scale (frequency) relationship compare in Short Time Fourier Transform vs. Wavelet Transforms.

One major advantage afforded by wavelets is the ability to perform local analysis - that is, to analyze a localized area of a larger signal. Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss, aspects like trends, breakdown points, discontinuities in higher derivatives, and self-similarity.

In classical wavelet decomposition, the image is split into an approximation and details images. The approximation is then split itself into a second-level approximation and details fig (4) depicts this process in 3-level decomposition. For n-level decomposition, the signal is decomposed in the following way:

$$\begin{aligned}
 cA_n &= [L_x * [L_y * A_{n-1}]_{-2,1}]_{-1,2} \\
 cD_{n1} &= [L_x * [H_y * A_{n-1}]_{-2,1}]_{-1,2} \\
 cD_{n2} &= [H_x * [L_y * A_{n-1}]_{-2,1}]_{-1,2} \\
 cD_{n3} &= [H_x * [H_y * A_{n-1}]_{-2,1}]_{-1,2}
 \end{aligned} \quad (1)$$

where * denotes the convolution operator, $^{-2,1}(-1,2)$ sub-sampling along the rows (columns) and $A_0 = I(x, y)$ is the original image. cA_n is obtained by low pass filtering and is the approximation image at scale n. The details images cD_{ni} are obtained by band pass (or high pass) filtering in a specific direction and thus contain directional detail information at scale n. The original image I is thus represented by a set of sub-

images at several scales; $\{cA_n, cD_{ni}\}$ or $cA_k, [cD_k^H, cD_k^V, cD_k^D]$ as illustrated in fig (5).

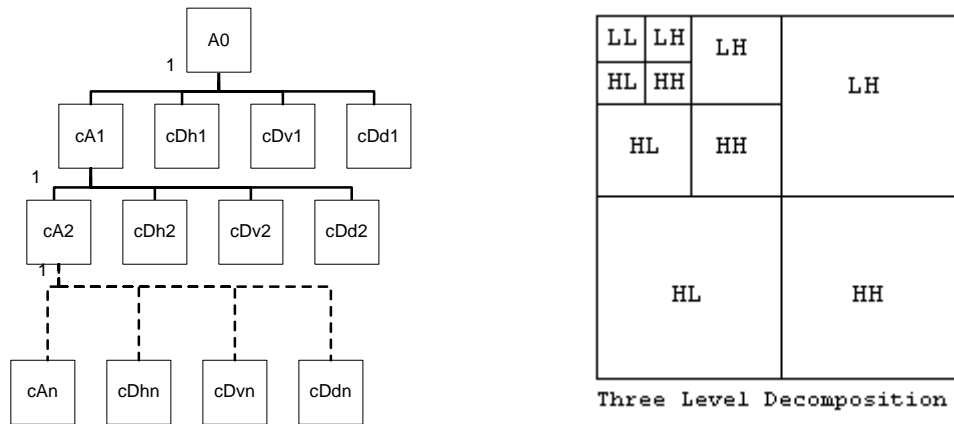


Fig (4) 3-level decomposition in 2D discrete wavelet transform.

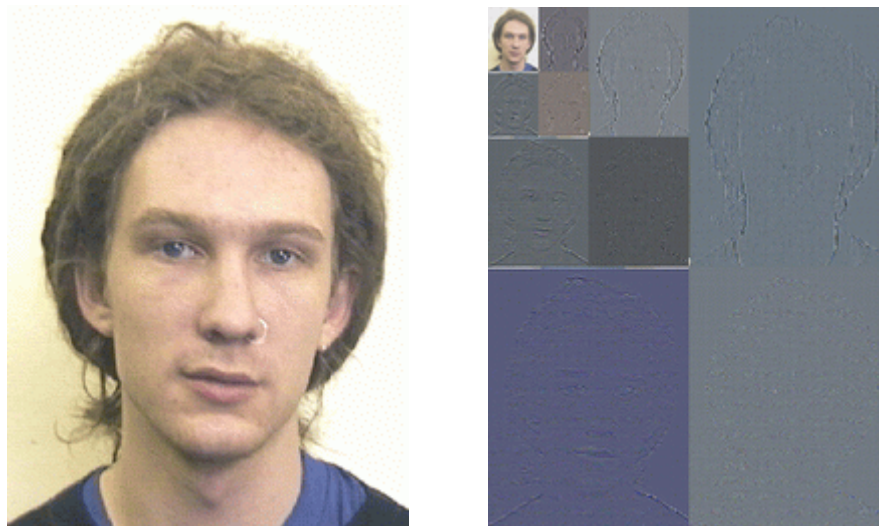


Figure 5, Stirling Database. David, 2-DWT, Daub6, 3 Level decomposition.

In Figures (4,5), the subband LL corresponds to the low frequency components in both vertical and horizontal directions of the original image. Therefore, it is the low frequency subband of the original image. The subband LH corresponds to the low frequency component in the horizontal direction and high frequency components in vertical direction. Therefore it holds the vertical edge details. Similar interpretation is made on the subbands HL and HH.

Feature vectors extraction

Before proceeding with wavelet analysis and feature extraction, we need to bring up two points, first; we aim at segmenting the face image in order to separate the face from the background. Since generally the background is simple and homogeneous, (i.e., dark, white ...) we apply a heuristic method for defining the rectangle that contains the face only, we start by defining the top-left and the right-bottom most pixels of non

background, then, a rectangular area (bounding box) containing the face is obtained. After this step of preprocessing, the 3 level wavelet decomposition is performed on the face bounding box. There is no need to perform a deeper decomposition because after the fifth level, the size of the image becomes too small and no more valuable information is obtained see [8, 9, 11]. At the third level of decomposition, we obtain one image of approximation (low frequency image) and 9 of details for each channel (RGB). Therefore, the face image is described by 30 wavelet coefficient matrices (wavelet transform output), which represent quite a huge amount of information (equal to the size of the input image!).

Second; It is also well-known that, the complexity of a classifier grows rapidly with the number of dimensions of the pattern space, so it is important to take decisions only on the most essential, so-called discriminatory information, which is conveyed by the extracted features, also one has to keep in mind the curse of dimensionality. Bellman showed that, with increasing of feature vectors the discriminating power decreases [12]. Thus, we are faced with the need of dimensionality reduction. Each of the 30 coefficient matrices contains information about the texture of the face and the special intensity change values. An efficient way of reducing the dimensionality and characterizing textural information and special intensity is to compute a set of moments. Thus, we decided to extract 9 measures from the low-resolution (low frequency) which constitute the mean value m_{out} of the $\{cA_5, cD_5^H, cD_5^V, cD_5^D, \mathbf{K}\mathbf{K}, cD_1^V, cD_1^D\}$ matrices for each channel RGB this corresponds to average intensity values of the face under certain lighting condition, and 81 variances S_{out}^2 of the matrices of red, green and blue channels which correspond to $\{cA_1, cD_1^H, cD_1^V, cD_1^D \dots cA_5, cD_5^H, cD_5^V, cD_5^D\}$ this represents the global value of the face texture, skin, facial hair, and facial expression. [13] Used the same approach on gray scale images (FERET Database) using 2 level wavelet packet decomposition. Thus we obtain a total of 90 moment vector for the next stage (usually the classification stage), which is quite reasonable compared to $326 \times 440 = 14344$ inputs in case the data base we are using consist of images with an average size of 326×440 . The mean and the variance are described as following:

$$m = \frac{1}{k} \sum_{i=1}^k f_i \quad (2) \quad \text{And} \quad s^2 = \frac{1}{k} \sum_{i=1}^k (f_i - m)^2 \quad (3)$$

and the vectors to be used as an input for the next stage are:

$$n = \mathbf{U}_{i=1, j=1}^{12,48} \{m_i, s_j^2\} \quad (4)$$

where m_i is the mean value for last level coefficient matrices, s_j^2 is the variance for the matrices of all level decompositions. In fact, its very much possible after the extraction of all the vectors of the face images database, to try keeping the most meaningful components by checking the mean and variance values of each of them for all the feature vectors. Only the components with a mean, variance values above a predefined threshold are considered for feature vector formation. Typically, in the case of using a post classification stage such as ANN, feature vectors with the largest value are built for threshold values of $\{-2.9, 2.9\}$.

To implement our proposed method on colored images, the first requirement is to understand the basic format of image files (like bmp, jpg, tiff ...), then if necessary applying some image enhancement or de-noising prior to applying the wavelet filters. What important is obtaining the RGBQUAD values of each pixel (point) in the array data representing the face in that image, then from these values we can form 3 float arrays for each channel representing the red, green and blue color values for the face's DIB section. Then we find the wavelet coefficients. The wavelet decomposition of these float data arrays is derived from 2-channel sub-band filtering with two filter sequences (as mention before) (h_k) , the smoothing or scaling filter, and (g_k) , the detail, or wavelet filter. These filters should have the following special properties:

$$\begin{aligned} \int_{-\infty}^{\infty} h_k^2 dk &= \sqrt{2} \\ g_j &= (-1)^j h_{1-j} \\ \int_{-\infty}^{\infty} g_k^2 dk &= 0 \\ \int_{-\infty}^{\infty} h_k h_{k+2m} dk &= d_{0m}, \text{ for all } m \end{aligned} \quad (5)$$

for more details about these condition see [14,15,16,17], for the multiresolution decomposition of the arrays we used orthonormal and biorthogonal wavelets which are extensions of orthonormal bases developed by Mallat and Meyer[18], thus the transform will be non redundant. We should note that even though most orthonormal wavelet bases come from multiresolution, not all do. The sufficient conditions for generating an orthonormal multiresolution analysis and the appropriate scaling functions:

_ Filter decay condition: $|h_k| = O(\frac{1}{1+k^2})$ (6).

_ $m_0(w) \neq 0$ for $w \in [-\frac{p}{2}, \frac{p}{2}]$ (7).

Then the product

$$\frac{1}{\sqrt{2^p}} \prod_{k=1}^{\infty} m_0\left(\frac{w}{2^k}\right) \quad (8)$$

converges to an L^2 function $f_{\neq}(x)$, and the function f generates a multiresolution analysis, various sufficient conditions can be found in [18,19,20] these conditions don't always guarantee that we actually get an orthonormal basis, or even that the key stability condition holds, nor do they guarantee that the scaling function f satisfying these conditions can be found (you should also check for the necessary and sufficient conditions for compactly supported orthonormal multiresolution in [18,19]). As for the Biorthogonal wavelets which are nothing but a orthogonal wavelets generalized by using two sets of functions, $f_{ij}, y_{ij}, F_{ij}, Y_{ij}$. The wavelets $(\psi_{ij}), (\Psi_{ij})$ don't form orthogonal bases but they are required to form dual bases for L^2 . Formal necessary condition for biorthogonality $m_0(w)\overline{m_0(w)} + m_0(w+p)\overline{m_0(w+p)} = 1$. This replaces the necessary orthonormality condition we had before, for more details see [21]. Following are the coefficients for some of the discrete wavelet analysis filters we

used for the multiresolution decomposition: the symmetrical extension wavelets like Daubechies filters ...

Daub4 = {0.4829629131445341, 0.8365163037378077, 0.2241438680420134, -0.1294095225512603};

Daub6 = { 0.3326705529500825, 0.8068915093110924, 0.4598775021184914, -0.1350110200102546, -0.0854412738820267, 0.0352262918857095 };

Haar = {1.0/Sqrt2, 1.0/Sqrt2};

Adelson= { 0.028220367, -0.060394127, -0.07388188, 0.41394752, 0.7984298, 0.41394752, -0.07388188,

-0.060394127, 0.028220367 }; //Filter from Eero Simoncelli's PhD thesis --used in Edward Adelson's EPIC wavelet coder. These are probably the filter coefficients used in Shapiro's EZW paper. Provided by Geoff Davis.

And the non symmetrical or periodical extension wavelet filters ...

Antonini={3.782845550699535e-02,-2.384946501937986e-02,-1.106244044184226e-01, 3.774028556126536e-01,8.526986790094022e-01,3.774028556126537e-01,-1.106244044184226e-01,

-2.384946501937986e-02,3.782845550699535e-02 }; //Filter from M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using wavelet transform", IEEE Transactions on Image Processing", Vol. pp. 205-220, 1992. Provided by Geoff Davis.

Brislawn= { 0.037828455506995, -0.023849465019380,-0.110624404418423, 0.377402855612654,0.852698679009403,0.377402855612654,-0.110624404418423,-0.023849465019380, 0.037828455506995 }; //Filters from Chris Brislawn's tutorial code

Now we use the above filters to compute the standard decomposition [22] of the face images by first applying (convoluting) the above filters to each row of pixel values. This operation gives us an average value along with detail coefficients for each row. Next, we treat these transformed rows as if they were themselves an image, and apply above filters to each column pixel value; we then repeat the whole procedure for the desired number of levels. The resulting values are all detail coefficient arrays except for a single overall average coefficient array, as illustrated in Figs 4, 5. For more info on how to convoluting images with given filters see [23, 24]. Next, as mentioned before we reduce the dimensionality of the outcome coefficient arrays, by computing the mean

m_{out} and variance S_{out}^2 for each coefficient array.

Feature Extraction System

In order to show the good representational scheme (robustness) and the computational efficiency of our proposed method we performed some experiments using test face images from the Stirling University colored face database [25]. These images are with an average size of 326x440 pixels (which is quite large), and with different facial expressions, lighting conditions, and different poses.

Our scheme consists of two stages. In the first stage we will consider the representational aspect, that is, we'll find the face only bounding rectangle in the image, and in order to achieve this we developed the following heuristic (for Stirling faces database), which consists of 4 parts:

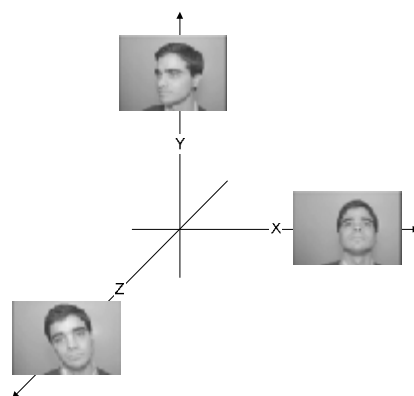
First we apply the softening filter to eliminate minor lighting variations (keep in mind that the proposed scheme is for faces under stable conditions), then we apply the following procedure:

```

for(DWORD j=0; j<NumofColorsInImage; j++){
    BYTE i=(BYTE)(j *(255/( NumofColorsInImage -1)));
    RGBQUAD HPV={ 120,240,i,0};//HPV corresponds to Hue Palette
    Values SetPaletteIndex((BYTE)j,HSLtoRGB(HPV));
}

```

this procedure yields a pseudo-color image that makes it very easy depending on the blue and red channel values (purple levels, shown in fig7) to define the face only bounding rectangle. Second we try to locate the center of main purple pixel concentration in the image in order to locate the preliminary horizontal center X_c , we achieve that by scanning the image horizontally from both right and left (in pairs) with predefined vertical step sizes until a certain continuous of purple pixels (red>220, blue>220) occur, then we calculate the mean of mid pair occurrences and make it X_c . Third we try to locate the first occurrence of the vertical forehead purple cluster Y_c , and lastly, by means of ellipse template matching we define the new X_{cc} and Y_{cc} and the best angle (checking whether the face is rotated about Z axis, we consider Y axis rotation to be a non frontal case but a profile case, while as the X axis rotation to be a view tolerant case. See fig6), thus we can define the face only bounding rectangle regardless of whether it's translated (not centered) or rotated.



Fig(6).The direction of rotation.

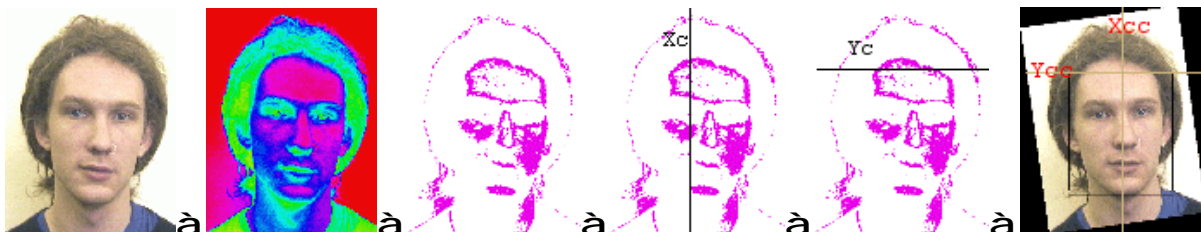


Fig (7) A Sample of resized colored faces in Stirling database and the heuristic used in defining the bounding rectangle of the face only.

In the second stage we apply the multi-resolution decomposition using one of the previously mentioned quadrature mirror filters (Duab6, villa6, ...), the decomposition level is highly experimental and case dependent, usually an entropy-

based criterion is used to determine the best decomposition level, for our case study and depending on our face database image sizes we found that 3 level decomposition is quite sufficient. We must remember that these coefficient matrices represent the different directional variation response of the RGB channels pixel color values to the currently selected wavelet, scaling functions (H, L filters), and that the first level sub-bands have more coefficients and they give a more precise variation description depending on variation position in the image and on the size of this variation in that image, however the huge number of these coefficients in those first level sub-bands makes it impractical to directly utilize them, thus, what we tried to achieve is to locate the corresponding coefficients for the most reliable and non-changeable parts of the face due to facial expressions, sizing (chin, nose, eye sockets, and forehead regions) fig (8) shows the four regions which are defined by pre-sized shrinking rectangles centered around X_{cc} and starting from Y_{cc} with 5 pixels in between except for the last region which is calculated from the bottom of bounding rectangle of the face.

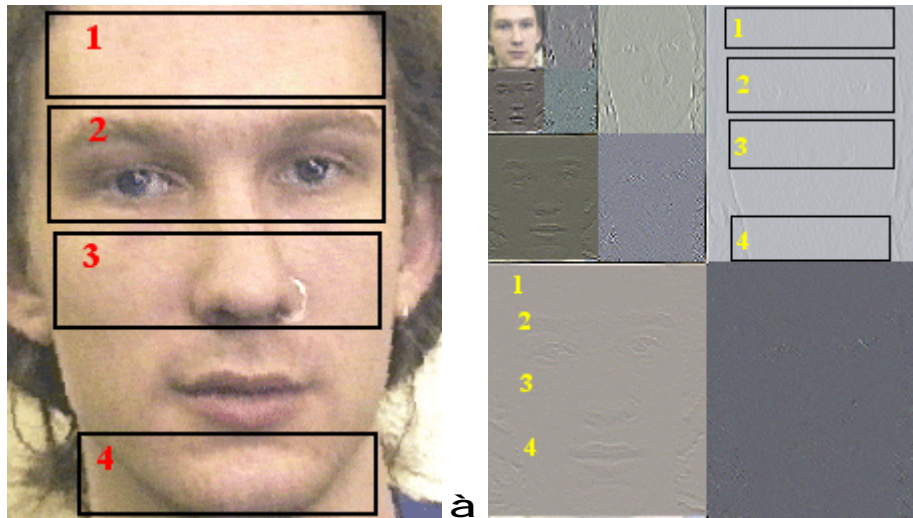


Fig (8) A sample image and its corresponding 3-level wavelet decomposition and the predefined regions from which we calculate the variance and mean feature vectors (only numbered in cDh_1 , cDV_1 for visual clearance).

From the previous regions we calculate 90 vectors as following:

- § For the first region (1) which corresponds to forehead (invariant to facial expressions, but maybe obscured by facial hair) we calculate the variance and mean value for the cA_3 (approximation subband) coefficients, and the variance for the LH, HL subbands in all levels which is 6 for each color channel, thus the total is $2 \times 3 + 6 \times 3 = 24$ feature vector describing the horizontal, vertical edge details and texture in this region.
- § For region (2) which corresponds to eyes and their surrounding muscles (this area is somehow expression dependent) we calculate the coefficient variance in all LH, HL subbands for each channel, thus the total is $6 \times 3 = 18$ feature vector describing only the horizontal, vertical edge details.
- § For the third region (3) corresponding to the nose and upper cheek muscles (highly invariant in different facial expressions) we do the same as for region(1), that is we obtain 24 vectors.

§ The last region (4) is the chin and lower face region (highly invariant in facial expressions) again we calculate 24 feature vectors.

Next we want to evaluate the computational power of our method, the face boundary definition is linear with N (where N is number of image pixels). The computational complexity for 3 level multi-resolution decomposition is also linear with M (the number of computed coefficients) which is equal to N that is $O(N)$ quite efficient while best other transformation, also in their fast implementation, lead to $M \cdot \log_2(M)$ complexity. Both mean and variance computation complexity is linear with N.

Experimental Results and Discussion

Detecting and locating the face in a face feature extraction or recognition system is equally important and difficult. In the first stage of our proposed scheme we think that we effectively achieved locating the face only region under some conditions, following are the discussion of these conditions and their effect on our heuristic. First we discuss the effect of translation; our heuristic can locate precisely the horizontal center and the vertical start pose of the forehead if only the face is located in a position in which there is at least 10 pixels of non face texture like background as illustrated below:



Fig9. Different translated faces and the corresponding bounding rectangles, from left to right:
 1. right shifted, good 2. left shifted, good
 3. right and up shifted, good 4. right and down shift, good



continue Fig 9. Translated face with no minimum 10 pixel background.
 1. no right 10 pixel, bad 2. no left 10 pixel, bad 3. no left-up 10 pixel, bad

now we discuss the rotational effect of the faces, making our scheme invulnerable to different Y axis rotations is the most CPU (serial) time consuming part of the heuristic, therefore we limited the rotation detection range to -15,+15 degrees, figure 10 shows some rotated sample images.



Fig 10. Different rotated face. 1. 8 degrees 2.10 degrees 3.-10 degrees

For faces rotated more than 15 degrees left or right our heuristic will simply fail to locate the true bounding rectangle.

Next we discuss the stage 2 invulnerability to different facial expressions or to partial covering of one face area. As we showed in the Feature Extraction System section we have 4 region representing the eyes, chin, nose, and forehead ; and for each region we compute vectors from the horizontal feature sensitive subbands (coefficient matrices) HLs, vertical feature sensitive subbands LHs and from the insensitive feature subbands LLs of all 3 levels of the wavelet multiresolution decomposition. In case of a partial covering of one area, the system can still provide the exact feature vectors from the other non obscured regions (of course its up to the classification system to evaluate this) fig11. shows some of covered face regions and where our system can still provide good bulk of information.

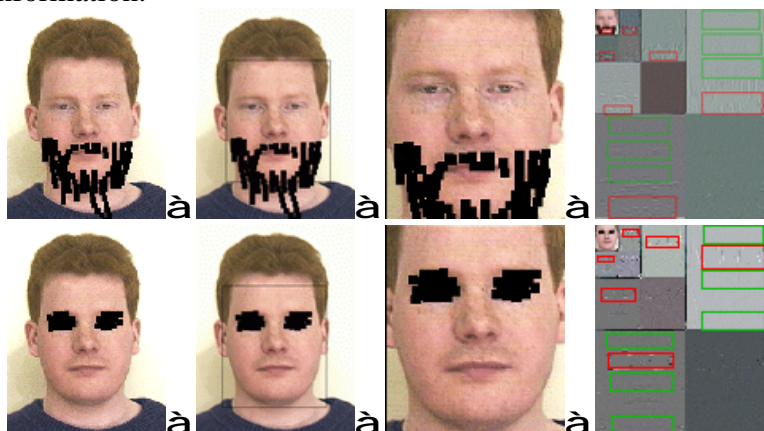


Fig 11. Some modified face regions (covered) and the outcome of the feature extraction system, green rectangles refer to unaffected valid vector extraction region while as red rectangles refer to invalid regions.

Now we'll discuss the effect of different facial expressions. The change of facial expressions mainly varies in eyes, mouth and other face muscles, and from technical point of view, it involves mainly changes of edges, that is facial expression like smiling will have an effect (as a shift, or textural change) around the mouth and eye's muscles but other area of the face won't change much, so we can still consider our system to be effective as in the previous case of regional obscuring of some regions of the face, plus the fact the we calculate the variance and mean as global measure of the texture and intensity of that region in each subband (except the structural subband HH), and as we

move into higher subbands (cA_3 , cDh_3 , cDv_3 , ...) the different facial expression will be less evident and effective.

Conclusion

Our experiments show that this proposed method is quite efficient under stable conditions like small facial expression, pose, rotation and illumination changes. The mean value vectors used relate to self similarities of an image (same image but with noise or modification), while the variance related vectors provide some invulnerability against lighting, transformation and scaling changes. The computational efficiency of this method is very good compared for other feature detection and representational methods like the edge-chain code, structural-grammar (local feature detection methods), or template matching methods, if large image sizes were to be used, it's also much better than other alternative methods like the Neural Networks which have the advantage of learning from representational examples, but if used merely they can be very computationally expensive, hard to train, and sensitive to changes in expression, orientation and posing. If we use a neural network as a later classification stage the overall system becomes more robust and precise. Finally we must mention the great capability of wavelet if they are used for high end facial matching systems, and its capability to remove redundant data (image compression).

Acknowledgment

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